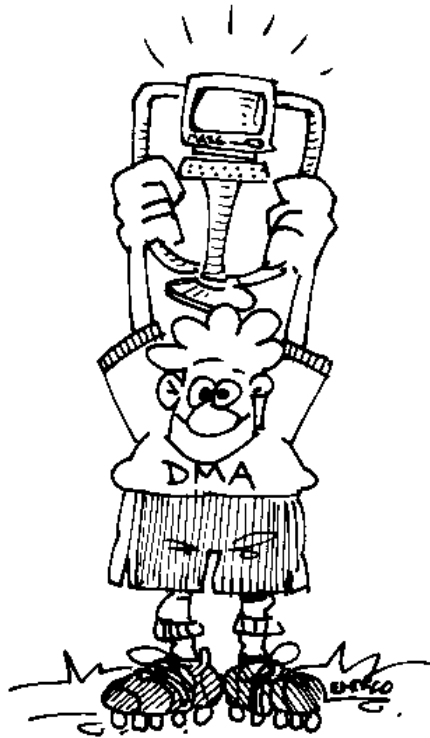




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Statistical Models for Knock-out Soccer Tournaments



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1 Introduction

1.1 Preliminary

Sports events and tournament competitions provide excellent opportunities for model building and using basic statistical methodology in an interesting way. In this paper, a logistic regression model using seed positions (conceived through a seeding coefficient) is applied to European soccer Cups tournament data in order to predict the probability of winning the tournament for each one of the participating teams, and the predicted probabilities of each team reaching a certain leg such as the quarter final.

1.2 European Cups

This section was coined by M. Protzen (1996).

There are four different European Cups

- the Champions Cup (CC),
- the Cup Winners Cup (CWC),
- the UEFA Cup (UC), originally intended for League runners up,
- and the UEFA Intertoto Cup (UIC).

The qualification for these competitions depends on the performance in National Leagues Cup competitions respective. The Champion usually enters the CC, the Cup Winner (losing finalist in case of a double respective) enters the CWC, while a variable number of League runners up enter the UC (in some countries winners of a League Cup enter the UC as well). Teams finishing below those qualifying for the UC may enter the UIC.

1.2.1 Champions Cup

Originally this Cup was intended for the league winners of the member FAs of UEFA. A while ago the qualification process for the CC has been modified such that only the CC defender together with the top 23 champions enters the CC while all other champions will enter the UC. The CC defender and the 7 best champions advance directly to the league stage while the champions ranked 8th to 15th have to play a preliminary tie against one of the champions ranked 16th to 23rd. To determine the top champions (and the number of UC berths allocated to a country) UEFA maintains a ranking list.

New format for 1997/98

For the upcoming season UEFA has once again changed the format to allow the vice champions of the 8 top ranked countries as well as the champions of up to 48 countries into the competition.

The league stage will be played in six groups of four teams. From there the six group winners together with two best runners-up will advance to the quarter finals. The two best runners-up will be determined by points achieved, goal difference, goals scored and goals scored away.

To cut down the number of teams the champions of countries ranked 17 to 48 in the most recent ranking table play a preliminary round in late July. For the losers of this tie their European campaign is over. Winners play in the qualification round together with champions of countries ranked 9 to 16 and vice champions of countries ranked 1 to 8. Winners of this round advance to the league stage where they will be joined by the champions of the top ranked countries. Losers will play in the first round of the UEFA Cup.

The defending Cup Winner gets an automatic place in the league stage regardless where he finishes in his national competition. This may mean that the champions of the countries ranked 8th, 16th (and 48th) are 'downgraded'.

1.2.2 Cup Winners Cup

The Cup Winners of the FA Cups of the member FAs of UEFA get to play in this Cup unless they also win the domestic championship (in which case they will be replaced by the losing cup finalists). The CWC defender is also eligible to play (unless he becomes national champion).

UEFA is also considering to allow a second team from the eight top ranked countries to play the CWC. How the second team will be determined has not yet been fixed. This modification will not effect for 96/97 and 97/98 competitions.

1.2.3 UEFA Cup

The UEFA Cup started (under the name "Fairs Cup") as a competition between city teams from cities which host a trade fair. Later it was a competition for the runners up of the various leagues, some countries enter also their League Cup winner. The number of participants from each country is determined by the UEFA ranking list. Since the Champions League was introduced the champions not eligible for the Champions League are entered into the UEFA Cup as well, another three berths are reserved for the Inter-toto Cup semi finalists and three are given to countries who top the UEFA Fair Play competition.

- 79 participants are determined by the UEFA coefficient table:
 - 3 countries with four berths
 - 5 countries with three berths
 - 13 countries with two berths
 - 26 countries with one berth
- 24 champions not entering the CC
- 3 Intertoto Cup participants
- 3 Fair Play competition winners

The three countries which top the UEFA Fair Play competition each get one additional UEFA cup berth in the next competition. The 1995/96 berths were allocated to Norway, England and Luxembourg, the 1996/97 berths have been awarded to Sweden, Russia and Finland (one berth originally awarded to England has been revoked because of fielding youth and reserve teams in the 1995 UIC) and in 1997/98 Norway, England and Sweden will get the berths.

- 1 defending UEFA Cup Holder (only if they do not qualify for any European competition otherwise)
- 8 preliminary round losers from the Champions Cup (since 1996/97)

Altogether, there are 118 participants.

For the *96/97 competition* the following format will be used:

- Preliminary round in July, involving 54 teams from countries with the lowest country coefficient in the 95/96 ranking
- Qualifying round in August, involving 50 teams (27 winners from the qualifying round, 23 teams from the countries with low coefficients which did not play in the preliminary round) + 6 Intertoto teams.
- First round in September, involving 64 teams (25 winners from the preliminary round, 8 teams eliminated from the Champions Cup, 28 teams from countries with high rankings and 3 teams qualifying through the UIC).

Barring participation of teams from San Marino, Andorra and Bosnia the 97/98 competition will have 16 participants fewer: 24 champions of low ranked countries will not enter the UC but 16 instead of 8 CC qualification round losers will play in the first round.

From *1997/98 on* the final will be decided in a single game on neutral ground.

1.2.4 UEFA Intertoto Cup

The Intertoto Cup started as a way to guarantee income to the Toto-industry in the early sixties (in fact, this generated a significant sum of money for clubs in several countries, one of them the Netherlands). The first one or two seasons the group winners played out a final stage to determine an overall winner (the first being Ajax). After a few years the competition degenerated into summer practice for mid table teams of North, Central and East European countries. Teams from the British Islands, Spain or Italy rarely participated. No final stage was played after the initial (one or two) season(s) and nobody really cared for it. This new directive by UEFA seems like a final attempt of attracting some attention to it.

The 'new' UEFA Intertoto-Cup has started in the summer of 1995. The cup will be played during the summer, and the teams allowed to participate should have been the best finishing teams in the respective leagues the previous season not already qualified for any of the other three European Cups. The number of teams from each country is determined by the UEFA ranking of countries just as in the UC (with the exception that also the bottom clubs are entitled to one berth).

60 teams are divided into 12 groups with the winners of each group qualifying for a knock-out round, from where the best 3 teams qualify for the first round proper of the UEFA Cup the following autumn.

From the 1995 UIC two French teams, Racing Strasbourg and Girondins de Bordeaux, emerged successful and participated in the UC, Bordeaux even reached the UC final.

A number of changes have been announced for the 1996 UIC: The number of UC berths available through this competition has been raised to 3, Italian teams will join the competition and teams are urged to take the competition serious, i.e. play with their first team.

Some countries do not take part in the competition: England, Portugal, Scotland and Luxembourg. Italy sends only three teams, Spain and Greece only one. Italy and Spain later renounced their remaining berths for the UIC, they were heavily fined for that. Also Greece and Albania did not take their berths. Those six berths went to Estonia, Lithuania, Austria, Turkey, Denmark and Sweden.

Also the format of the cup has changed slightly: Again there will be 12 groups with 5 teams each. But fortunately the concept of "best seconds" has been cancelled: The 12 winner will play one elimination round with the 6 winners playing another round as part of the UC preliminary round.

In 1996 Silkeborg IF, Karlsruher SC and EA Guingamp qualified for the UC.

2 Descriptive Data Analysis

2.1 Data Structure

As said in the introduction European soccer Cups tournament data is used. We only considered the data from the Champions Cup (CC), from the Cup Winners Cup (CWC) and from the UEFA Cup (UC). The typical tournament structure does not begin until after the preliminary round(s), so we did not consider them. In this context the structure of an ordinary data looks like this:

```

leg  team.a  country.a  team.b  country.b  s.a.1  s.b.1  s.a.2  s.b.2
Round1 FKAustria(Wien)  Aut  DinamoMinsk  Bls  1  2  0  1
Round1 Milan  Ita  ZaglebieLubin  Pol  4  0  4  1
...
Round1 SpartakVladikavkaz Rus  Liverpool  Eng  1  2  0  0
Round1 ZimbruChisinau  Mol  RAYelgava  Lat  1  0  2  1
Round1 RodaJC(Kerkrade)  Ned  SCTOlimpija(Ljubljana) Slo  5  0  0  2
Round2 BrondbyIF  Den  Liverpool  Eng  0  0  1  0
...
Round2 SpartaPraha  Tch  ZimbruChisinau  Mol  4  3  2  0
Round2 GirondinsBordeaux  Fra  RotorVolgograd  Rus  2  1  1  0
Round3 PSV(Eindhoven)  Ned  WerderBremen  Ger  2  1  0  0
Round3 BayernMuenchen  Ger  SLBenfica  Por  4  1  3  1
...
Round3 Sevilla  Esp  FCBarcelona  Esp  1  1  1  3
Round3 SlaviaPraha  Tch  RCLens  Fra  0  0  1  0
Quarter FCBarcelona  Esp  PSV(Eindhoven)  Ned  2  2  3  2
Quarter SlaviaPraha  Tch  Roma  Ita  2  0  1  3
Quarter Milan  Ita  GirondinsBordeaux  Fra  2  0  0  3
Quarter BayernMuenchen  Ger  NottinghamForest  Eng  2  1  5  1
Semi BayernMuenchen  Ger  FCBarcelona  Esp  2  2  2  1
Semi SlaviaPraha  Tch  GirondinsBordeaux  Fra  0  1  0  1
Final BayernMuenchen  Ger  GirondinsBordeaux  Fra  2  0  3  1

```

where $s.a.1:s.b.1$ is for example the score of the 1st leg. This list represents the data from the UEFA Cup 1995/96.



2.2 Tree Structure

An example of a typical tournament structure is represented by figure 1. The tree structure is cognizable. In this section we want to describe how we obtained the tournament tree through the data. So let us have a look at figure 1. We always start with the winner of the tournament, in this case it is *A*. Then we will look for the team *A* played in the previous round, that would be in this case *C*. In the future *C* is marked. Now we are looking for the team *A* played against before playing against *C*, that would be *B*. We realize that the game *A* against *B* had be played in the first round of the tournament, so we are going back to the last marked team, that would be *C*, and so on.

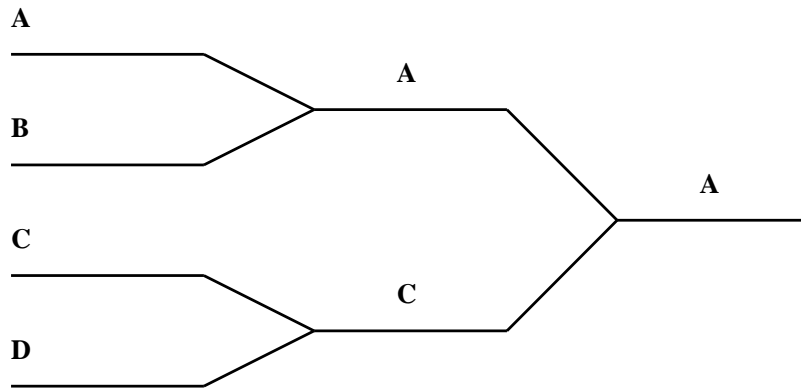


Figure 1 *Example of a typical tournament structure.*

For this reason we have programmed a Splus routine titled `pairings.tree`. In applying this routine to the Cup Winners Cup of the year 1989/90 we obtain figure 2.



cwc.8990



Figure 2 *The Cup Winners Cup 1989/90.*

In the appendix you will find the tournament trees of all European Cups considered.

2.3 Seeding Coefficients

To obtain the seed positions we calculated the seeding coefficients with the help of a ranking list. The basis for this ranking is the performance of teams in the three major European Cups during a three year period. Each team gets two points for a win and one point for a draw. One bonus point is allocated for reaching the quarter final, the semi final and the final. This procedure is similar to the one the UEFA uses for their ranking list. In the first time we calculated a coefficient which will be replaced later on, by a more appropriated coefficient: A weighted coefficient. Both coefficients are based on a list containing the performances during an European Cup. An example is the following list which shows the points achieved for the participants during the Champions Cup of the year 1995/96:

THE TABLE FOR cc.9596

	team	country	win	draw	lost	scored	against	points
1	Ajax	Ned	8	3	1	22	3	22
2	Juventus	Ita	6	3	3	22	9	18
3	Panathinaikos	Gre	5	3	2	11	6	15
4	SpartakMoscow	Rus	6	1	1	17	8	14
5	FCNantes	Fra	4	4	2	14	12	14
6	RealMadrid	Esp	4	1	3	12	7	10
7	BorussiaDortmund	Ger	2	3	3	8	11	8
8	LegiaWarsaw	Pol	2	2	4	5	11	7
9	FCPorto	Por	1	4	1	6	5	6
10	SteauaBucharest	Rom	1	2	3	1	5	4
11	RosenborgBK	Nor	2	0	4	11	16	4
12	Rangers	Sco	1	2	3	6	13	4
13	Ferencvarosi	Hun	1	2	3	9	19	4
14	BlackburnRovers	Eng	1	1	4	5	8	3
15	AaB(Aalborg)	Den	1	1	4	5	11	3
16	GrasshopperClubZuerich	Sui	0	2	4	3	13	2



2.3.1 First Approach

In this approach we calculated the performances of the teams over a three year period and summarized them into a list. The following list shows the points achieved for all teams who participated in an European Cup from 1992 to 1995.

	team	country	win	draw	lost	scored	against	points
1	Milan	Ita	23	8	4	55	10	60
2	Juventus	Ita	22	5	5	68	22	56
3	Parma	Ita	18	8	6	37	15	53
4	Ajax	Ned	17	6	3	47	14	45
5	ParisSaintGermain	Fra	15	6	7	39	19	42
6	BorussiaDortmund	Ger	16	4	10	43	30	42
7	SLBenfica	Por	13	7	3	43	23	37
8	FCPorto	Por	14	5	8	43	20	35
9	Arsenal	Eng	11	7	2	35	15	35
10	FCBarcelona	Esp	11	7	6	42	27	32
11	SpartakMoscow	Rus	12	6	5	46	31	32
12	RealMadrid	Esp	11	4	5	40	23	28
13	EintrachtFrankfurt	Ger	10	4	6	35	14	26
14	BayerLeverkusen	Ger	9	4	3	41	19	25
15	IFKGothenburg	Swe	10	3	5	26	18	24
16	AJAuxerre	Fra	8	5	5	37	21	24
17	OlympiqueMarseille	Fra	10	3	2	26	9	24
18	Internazionale	Ita	10	1	3	23	11	24
19	RSCAnderlecht	Bel	7	9	5	31	24	23
20	RealZaragoza	Esp	9	1	6	31	17	22
21	ClubBruggeKV	Bel	9	2	5	19	15	21
22	Boavista	Por	7	5	4	22	11	20
23	Galatasaray	Tur	6	7	9	17	29	19
24	Lazio	Ita	8	2	2	15	7	19
...								
211	KSVWaregem	Bel	0	0	2	1	6	0
212	DACDunajskaStreda	Tch	0	0	2	0	4	0
213	Motherwell	Sco	0	0	2	0	3	0

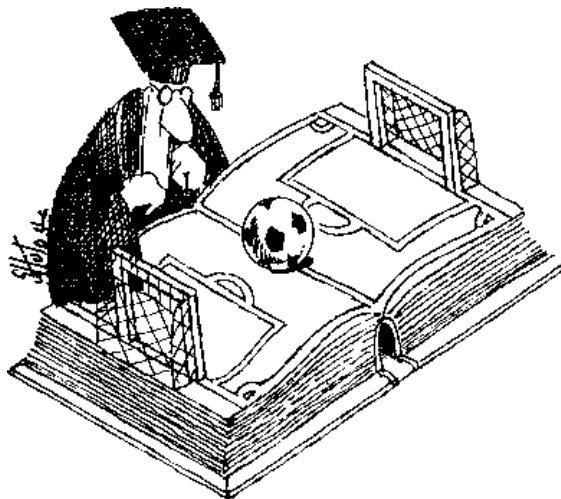
This ranking list over a three year period is now used to calculate the seeding coefficients. We defined the coefficient in the following way:

$$\text{Coefficient} = \frac{\text{Points achieved during the three year period}}{\text{Games played during the three year period}}$$

This is different from the UEFA coefficients, because the UEFA uses the sum of the ratio points achieved over games played for each of the five past years. An example of our calculations is the following list which shows the seeding coefficients for the UEFA Cup of the year 1993/4.

	team	country	coeff
1	Juventus	Ita	2.4000000
2	AtleticoMadrid	Esp	2.0625000
3	BayernMuenchen	Ger	1.9166667
4	AJAuxerre	Fra	1.8571429
5	BorussiaDortmund	Ger	1.7222222
6	SportingCP(Lisbon)	Por	1.6428571
7	Internazionale	Ita	1.6428571
8	BrondbyIF	Den	1.5000000
9	AstonVilla	Eng	1.5000000
10	RoyalAntwerp	Bel	1.4166667
11	Olympiakos(Pireus)	Gre	1.4000000
12	AdmiraWacker	Aut	1.4000000
13	Vitesse(Arnhem)	Ned	1.2500000
14	Trabzonspor	Tur	1.1666667
15	GirondinsBordeaux	Fra	1.1666667
16	EintrachtFrankfurt	Ger	1.1000000
17	SlaviaPraha	Tch	1.0000000
18	DinamoMoscow	Rus	1.0000000
...			
63	BSCYoungBoys	Sui	0.0000000
64	AaB(Aalborg)	Den	0.0000000

This approach was not satisfiable because there is a problem with teams which were playing in the Champions Cup. In the Champions Cup less games are played and so the teams will achieve less points. That is why their seeding coefficient would be smaller, even if they played in the most important European Cup. By calculating the probability for a team of winning the tournament for example (in using the program `prob.cte` (cf. section 3.2)) for the UEFA Cup 1995/96 we noticed that Girondins Bordeaux (Fra) had more chances to win the tournament than Barcelona (Esp). For every soccer fan this seems to be impossible, so for us. For this reason we did not use this approach to calculate the seeding coefficients.



2.3.2 Improvement

The improvement consists in calculating a weighted mean of the ratio points achieved over games played for each of the three past years, let be C_x this ratio for the year x . Hence we defined the coefficient for the year x as

$$\text{Coefficient} = \frac{3}{6}C_{x-1} + \frac{2}{6}C_{x-2} + \frac{1}{6}C_{x-3}.$$

With this choice of the weights we give most importance on the ratio of the previous year. If for example a team plays in a given year's UEFA Cup, but it did not play any European Cup for the last two years, the team will get a seeding coefficient (in using the previous method (see 2.3.1)) which seems to be too high because they played for the last time in the European Cup three years before - that is nonsense. This problem is absorbed by the improved coefficient because there will be the weight 1/6. That is why from now on this weighted seeding coefficient (briefly coefficient) is used. The following list shows the coefficient for the Cup Winners Cup of the year 1992/93:

	team	country	coeff
1	ASMonaco	Fra	1.44722222
2	AtleticoMadrid	Esp	1.33333333
3	SpartakMoscow	Rus	1.25000000
4	WerderBremen	Ger	1.21666667
5	Trabzonspor	Tur	1.00000000
6	SteauaBucharest	Rom	1.00000000
7	Feyenoord	Ned	0.91666667
8	AdmiraWacker	Aut	0.72222222
9	SpartaPrah	Tch	0.70833333
10	Glenavon	Nir	0.66666667
11	Boavista	Por	0.58333333
12	Liverpool	Eng	0.56250000
13	Parma	Ita	0.50000000
14	Olympiakos(Pireus)	Gre	0.50000000
15	RoyalAntwerp	Bel	0.37500000
16	FCLucerne	Sui	0.33333333
17	AvenirBeggen	Lux	0.33333333
18	Apollon(Limassol)	Cyp	0.33333333
19	Valur(Reykjavik)	Isl	0.25000000
20	ChernomoretsOdessa	Ukr	0.25000000
21	FCLevski(Sofia)	Bul	0.08333333
22	UjpestiTE	Hun	0.00000000
23	TPS(Turku)	Fin	0.00000000
24	MiedzLegnica	Pol	0.00000000
25	MariborBranik	Slo	0.00000000
26	HapoelPetachTikva	Isr	0.00000000
27	Hannover96	Ger	0.00000000

28	CardiffCity	Wal	0.00000000
29	Bohemians	Irl	0.00000000
30	Airdrieonians	Sco	0.00000000
31	AIK(Stockholm)	Swe	0.00000000
32	AGF(Arhus)	Den	0.00000000

Figure 3 shows the coefficient for different three year periods for some chosen teams. Let us have a look at the evolution of the coefficient of Juventus (Ita): Between the periods 1989/92 and 1990/93 the coefficient increased because they won the UEFA Cup in 1992/93 (yes, Juventus also wins the UEFA Cup not only the Champions Cup) in achieving 24 points. Between the periods 1990/93 and 1991/94 the coefficient decreased because in the 1993/94 edition of the UEFA Cup Juventus reached the quarter final in achieving in 8 games only 10 points (4 wins, 1 draw and 3 lost). And between the periods 1991/94 and 1992/95 he increased again because they won the UEFA Cup in 1994/95 again before becoming Italian champion. We also remark the constancy of the coefficient of Barcelona (Esp) who was the dominating team in European Cups during this periods.

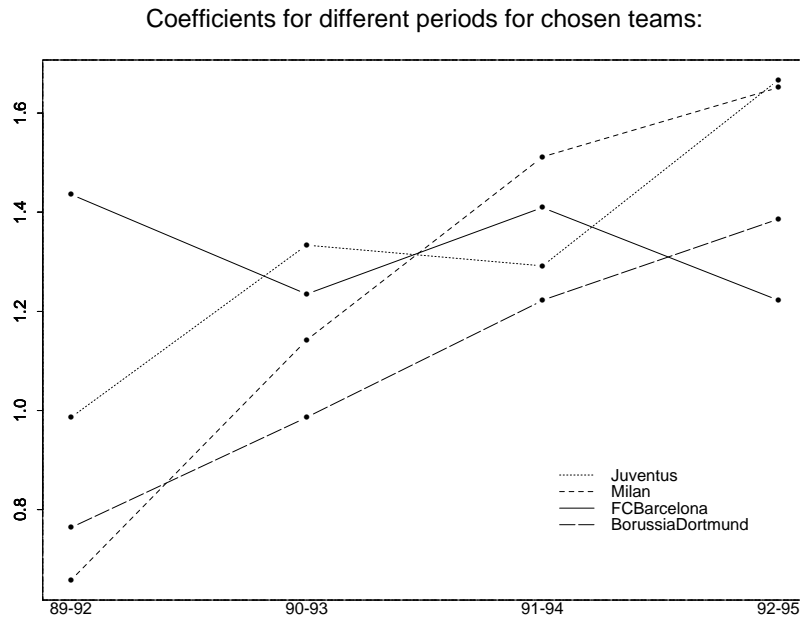


Figure 3 *The coefficient for different periods for some chosen teams.*

2.4 Tournament Analysis

2.4.1 Possible Sets of Winners

Predicting the probability of each seed winning the tournament requires the consideration of all possible paths and opponents. An example of a tournament is shown by figure 4.

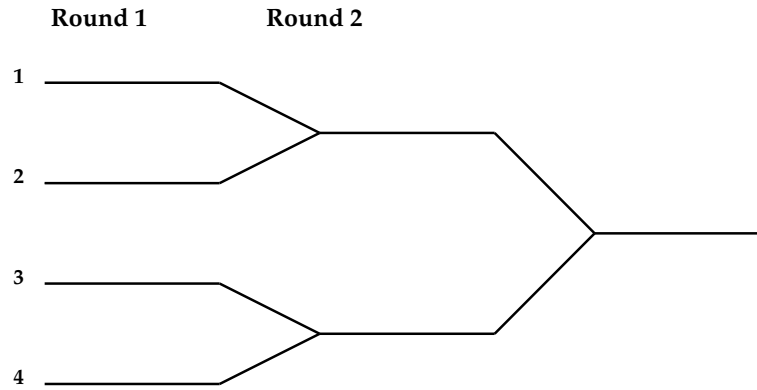


Figure 4 Example of a tournament.

Let us determine the teams that must be played for a team to become the champion. In Round 1 there are 2^2 possible sets of winners and in Round 2 there are 2 possible outcomes. Hence there are $2^2 \cdot 2 = 2^3$ possible outcomes for the 3 games.

By generalizing, we obtain the following table:

Round number	Games per round	Possible sets of winners
1	$n/2$	$2^{n/2}$
2	$n/4$	$2^{n/4}$
\vdots	\vdots	\vdots
m	$n/(2^m)$	$2^{n/(2^m)}$

where n is the number of teams participating in the tournament. From this it follows that in a tournament with n teams (and so $n - 1$ games) there are

$$\prod_{k=1}^m 2^{n/(2^k)}$$

possible sets of winners for the tournament, where m is the number of legs to play for reaching the final game.

Remark: In fact $m = \log_2 n$, because of the following relationship: $n = 2^m$.

2.4.2 Probability of Winning

Let $P_k(i, j)$ be the probability that team i wins against team j in the k th leg ($\forall i \neq j$). As in a tournament there is always a winner (i.e. there are any draws), it follows that $P_k(j, i) = 1 - P_k(i, j)$.

In the first round each team has only one possible opponent, but in the second round there are two possible opponents as shown in the previous section. That is why the probability analysis must include not only the probability of defeating each potential opponent, but also the probability of each potential opponent advancing to a particular game. To illustrate, let us have a look at figure 5.

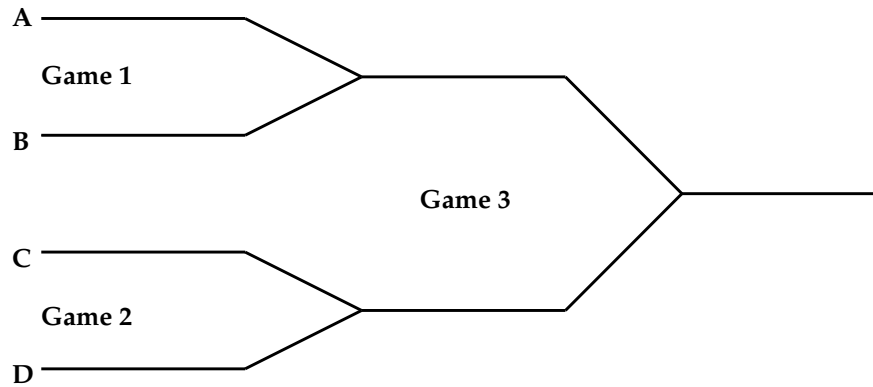


Figure 5 Example of a tournament.

Suppose that team A wins the tournament by defeating team B in game 1 and the winner of game 2 (C or D) in game 3, then the probability that team A wins the tournament would be :

$$\begin{aligned}
 P(A \text{ wins the tournament}) &= P(A \text{ wins in game 3}) \\
 &= P_1(A, B)P_2(A, \text{winner of game 2}) \\
 &= P_1(A, B)\{P_1(C, D)P_2(A, C) + P_1(D, C)P_2(A, D)\} \\
 &= P(A \text{ wins in leg 1})\{P(C \text{ wins in leg 1})P_2(A, C) + \\
 &\quad P(D \text{ wins in leg 1})P_2(A, D)\} \\
 &= P(A \text{ wins in leg 1}) \sum_{j \in \{C, D\}} P(j \text{ wins in leg 1})P_2(A, j)
 \end{aligned}$$

This probability is the probability of A winning the tournament and as well the probability of A winning in game 3, i.e. in leg $m = \log_2 n = \log_2 4 = \frac{\log 4}{\log 2} = 2$.

By generalizing, we obtain:

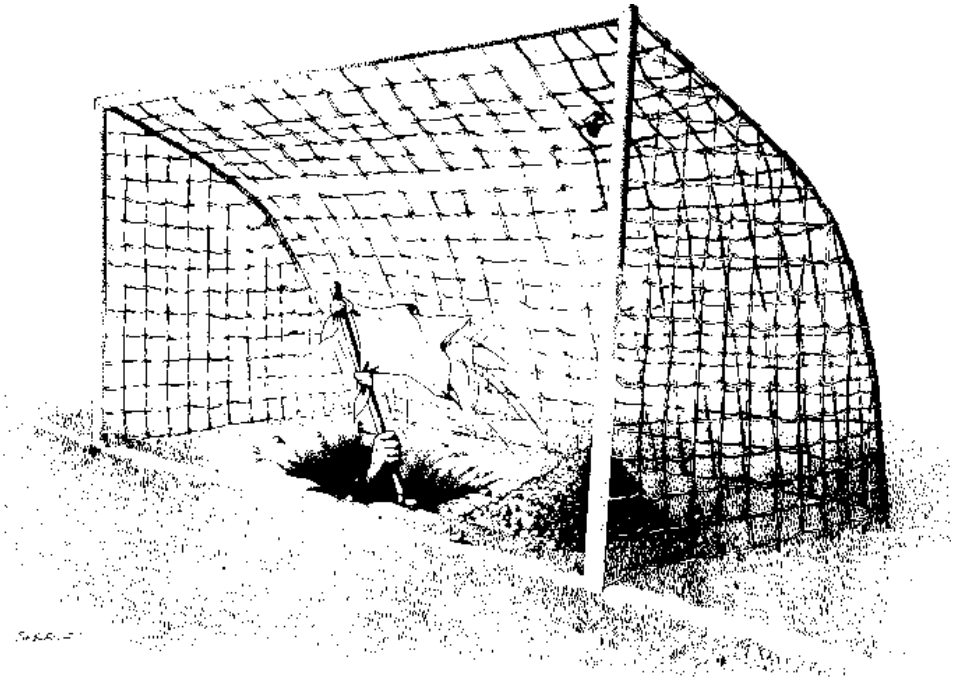
$$P(i \text{ wins in leg } k) = P(i \text{ wins in leg } k - 1) \sum_{j \in \mathcal{J}} P(j \text{ wins in leg } k - 1) P_k(i, j) \quad (1)$$

for $k = 2, \dots, \log_2 n$. \mathcal{J} is the set of all potential opponents of i for leg k .

Hence:

$$P(i \text{ wins the tournament}) = P(i \text{ wins in leg } k = \log_2 n) \quad (2)$$

We now must find probability models for determining $P_k(i, j)$.



2.4.3 Potential Opponents

By equation (1) we are forced to determinate the set of all potential opponents for a chosen team. For this reason we have programmed a function titled `potential.opponents`. The work of this function is illustrated by figure 6.

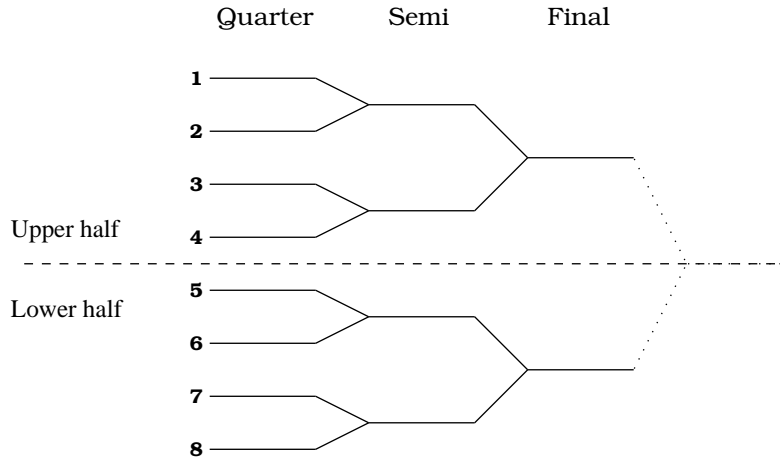
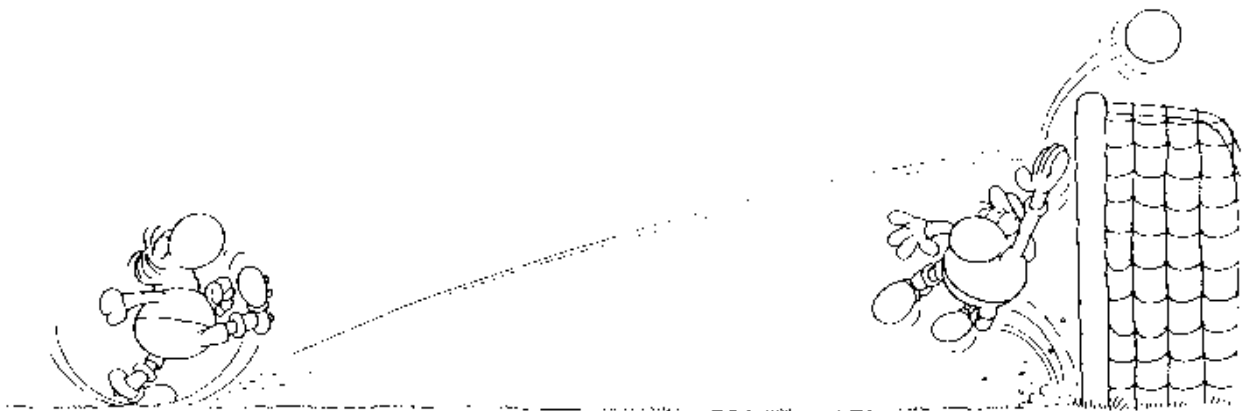


Figure 6 *Illustration of potential.opponents.*

In figure 6 there are eight teams participating in the tournament. If we want to determine all potential opponents for the final game of a team we only have to look if the team chosen is in the upper or in the lower half of the tournament tree. If the team is in the upper half the potential opponents will be all teams playing in the lower half, *i.e.* the following four teams: 5, 6, 7 and 8. In the same way it is possible to determine all potential opponents for the semi final: If the team is among the teams 3 and 4 the potential opponents are the teams playing in the upper quarter of the tournament tree, *i.e.* 1 and 2, and so on.



3 Statistical Modelisation

3.1 Logistic Regression Model

Schwertman *et al.* (1996) suggested in their work on probability models for the NCAA Regional Basketball Tournaments 11 different models for assigning probabilities of winning for each team in each individual game. These models were compared in three ways by using a chi-square statistic as a measure of the relative fit of the models. The chi-square values provided a measure of the relative accuracy of the various models. Hence the logistic model seemed to be the most satisfactory if the objective is to predict the winner of the tournament. That is the reason why we choose a logistic regression model. The model we assume is closely related to the Bradley-Terry model for paired comparisons (Bradley and Terry 1952). This relationship is shown in the appendix of this paper. Therefore the formula relating the seeding coefficients to winning probabilities is:

$$P_k(i, j) = \frac{e^{\alpha + \beta(S_k(i) - S_k(j))}}{1 + e^{\alpha + \beta(S_k(i) - S_k(j))}} \quad (3)$$

where $S_k(i)$ is the seeding coefficient of team i depending on leg k . Clearly we have $0 \leq P_k(i, j) \leq 1$ and $P_k(j, i) = 1 - P_k(i, j)$, $\forall k$ and $\forall i \neq j$.

These seeding coefficients represent the team strength.

3.2 Constancy of Team Strength

In the first time, we assume that the games are independent and that the seeding coefficients remain constant throughout the tournament. Hence $S_k(i) = S(i)$ and therefore $P_k(i, j) = P(i, j)$.

As each game has only two outcomes: A win or a loss, we are opposed to an independent Bernoulli trial. Let be Y_{ij} one Bernoulli trial, therefore:

$$Y_{ij} = \begin{cases} 1 & \text{if } i \text{ defeats } j \\ 0 & \text{else.} \end{cases}$$

It follows that $P(Y_{ij} = 1) = P(i, j)$. Hence the model (3) becomes:

$$E[Y_{ij}] = \frac{e^{\alpha + \beta(S(i) - S(j))}}{1 + e^{\alpha + \beta(S(i) - S(j))}}$$

where α and β are the parameters to estimate.

3.2.1 Model Chosen

The model is fit to a data set consisting of the game outcomes from season 1992/93 to 1995/6 in all three European Cups, *i.e.* the UEFA Cup, the Cup Winners Cup and the Champions Cup. It is important to notice that in the Champions Cup we took only the games of type knock-out, *i.e.* we did not take into consideration the league stage of the Champions Cup. In all we considered 442 game outcomes. In this regard it is important to say that even if two teams played twice against each other in a leg, we considered only the outcome of the leg, and not the outcomes of each game of the leg.

As response variable for the regression we took the following variable, let it be \mathbf{y} :

$$\mathbf{y} = \begin{bmatrix} y_{uc.9596} \\ y_{uc.9495} \\ y_{uc.9394} \\ y_{uc.9293} \\ y_{cwc.9596} \\ \dots \\ y_{cc.9293} \end{bmatrix}$$

where for example $y_{uc.9596}$ is a vector containing 1's and 0's. Let us have a look at two games of the UEFA Cup 1995/96:

leg	team.a	country.a	team.b	country.b	s.a.1	s.b.1	s.a.2	s.b.2
20 Round1	Glenavon	Nil	WerderBremen	Ger	0	2	0	5
21 Round1	BayernMuenchen	Ger	LokomotivMoscow	Rus	0	1	5	0

The 20th element of $y_{uc.9596}$ would be 0 because WerderBremen (*i.e.* team.b) won the game, and the 21th element of $y_{uc.9596}$ would be 1 because BayernMuenchen (*i.e.* team.a) won the game.

By fitting the model (3) we obtained the following estimations:

Estimations	Value	Std. Error	t value
$\hat{\alpha}$	-0.1305	0.1039	-1.2559
$\hat{\beta}$	1.2610	0.1608	7.8420

It is a good fit. This statement is validated by figure 7. One notices that this regression rejects the necessity of the intercept α . Therefore (3) becomes:

$$P(i, j) = \frac{e^{\hat{\beta}(S(i) - S(j))}}{1 + e^{\hat{\beta}(S(i) - S(j))}} \quad (4)$$

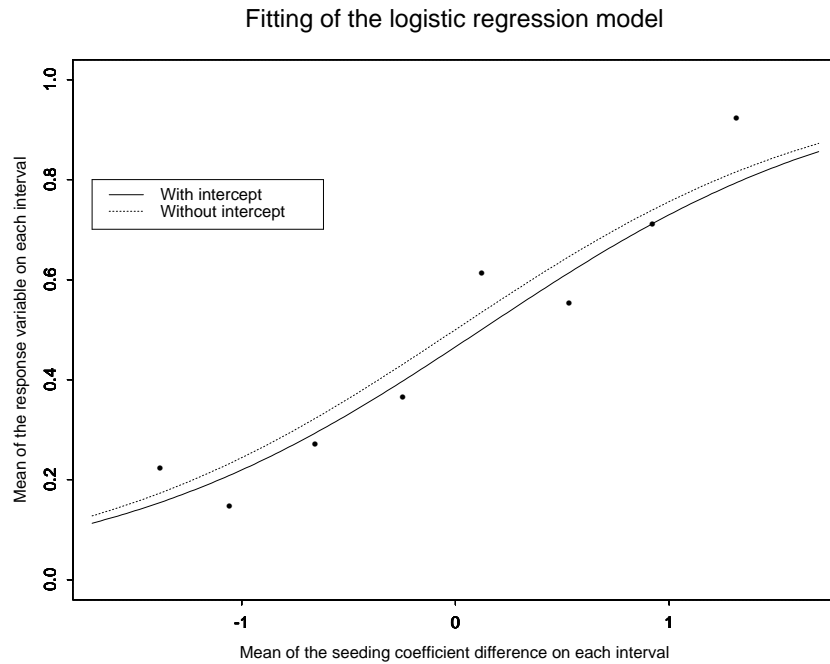
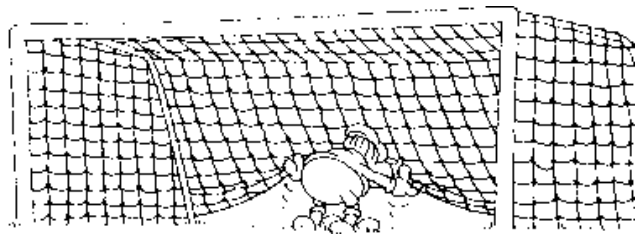


Figure 7 *Fitting of the logistic regression model. We noticed that the seeding coefficient differences lied between -1.8 and 1.8, therefore we divided this interval in 8 subintervals. In each of this subintervals we calculated the mean of the response variable and the mean of the seeding coefficient differences. One remarks that the curve without the intercept fits the observations best.*



3.2.2 The Program

With the help of equation (4) it is now possible to calculate the probability of each team winning against any other team (i.e. $P(i, j)$). In introducing these probabilities into equation (1) we are able to know the probability of each team winning any chosen leg and so, using equation (2), the tournament. For this reason we have programmed a function titled `prob.cte`. An issue of this function is shown by the following table who shows the probabilities for the Cup Winners Cup 1995/96:

	Round1	Round2	Quarter	Semi	Final
ParisSaintGermain	87.60	71.81	39.57	27.36	18.98
MoldeFK	12.40	4.97	0.95	0.24	0.06
Celtic	65.26	17.30	4.93	1.88	0.73
DinamoBatumi	34.74	5.93	1.13	0.28	0.07
Parma	89.01	79.22	49.19	35.38	25.51
KSTeuta(Durres)	10.99	5.50	1.12	0.28	0.07
HalmstadsBK	50.00	7.64	1.56	0.39	0.10
LokomotivSofia	50.00	7.64	1.56	0.39	0.10
RCDeportivoLaCoruna	68.37	32.21	15.36	4.69	2.21
Apoel(Lefkosia)	31.63	9.49	2.88	0.51	0.15
Trabzonspor	74.09	47.98	27.31	10.53	5.93
ZalgirisVilnius	25.91	10.33	3.36	0.65	0.21
RealZaragoza	76.19	40.65	21.63	7.39	3.81
InterZTSTBratislava	23.81	6.63	1.80	0.27	0.07
ClubBruggeKV	78.28	46.19	25.88	9.50	5.16
ShakhtarDonetsk	21.72	6.53	1.77	0.27	0.07
SKRapid(Wien)	55.24	16.78	6.17	1.98	0.39
PetrolulPloiesti	44.76	11.76	3.79	1.06	0.18
SportingCP(Lisbon)	71.32	54.96	34.67	19.76	8.18
MaccabiHaifa	28.68	16.50	6.86	2.50	0.58
DinamoMoscow	75.09	48.26	27.06	14.01	5.09
AraratErevan	24.91	9.49	2.98	0.83	0.14
SKHradecKralove	32.40	10.09	3.17	0.89	0.15
FCKobenhavn	67.60	32.15	15.29	6.63	1.90
Feyenoord	82.37	66.36	47.22	30.42	14.08
DAGLiepaya	17.63	8.31	2.99	0.88	0.15
Everton	44.76	10.45	3.76	1.10	0.18
KR(Reykjavik)	55.24	14.89	6.06	2.04	0.40
BorussiaMonchengladbach	50.00	15.06	4.18	1.23	0.21
SileksKratovo	50.00	15.06	4.18	1.23	0.21
AEK(Athens)	40.91	26.70	10.75	4.64	1.24
FCSion	59.09	43.18	20.87	10.82	3.69

Let us take for example Paris Saint Germain (Fra): Their chance to win in round 1 against Molde FK (Nor) is 87.6%, to win in round 2 against Celtic (Sco) or Dinamo Batumi (Geo) their chance is 71.8%, and to win the final, *i.e.* the tournament, their chance is about 19%. The real outcomes are shown by figure 8.

cwc.9596

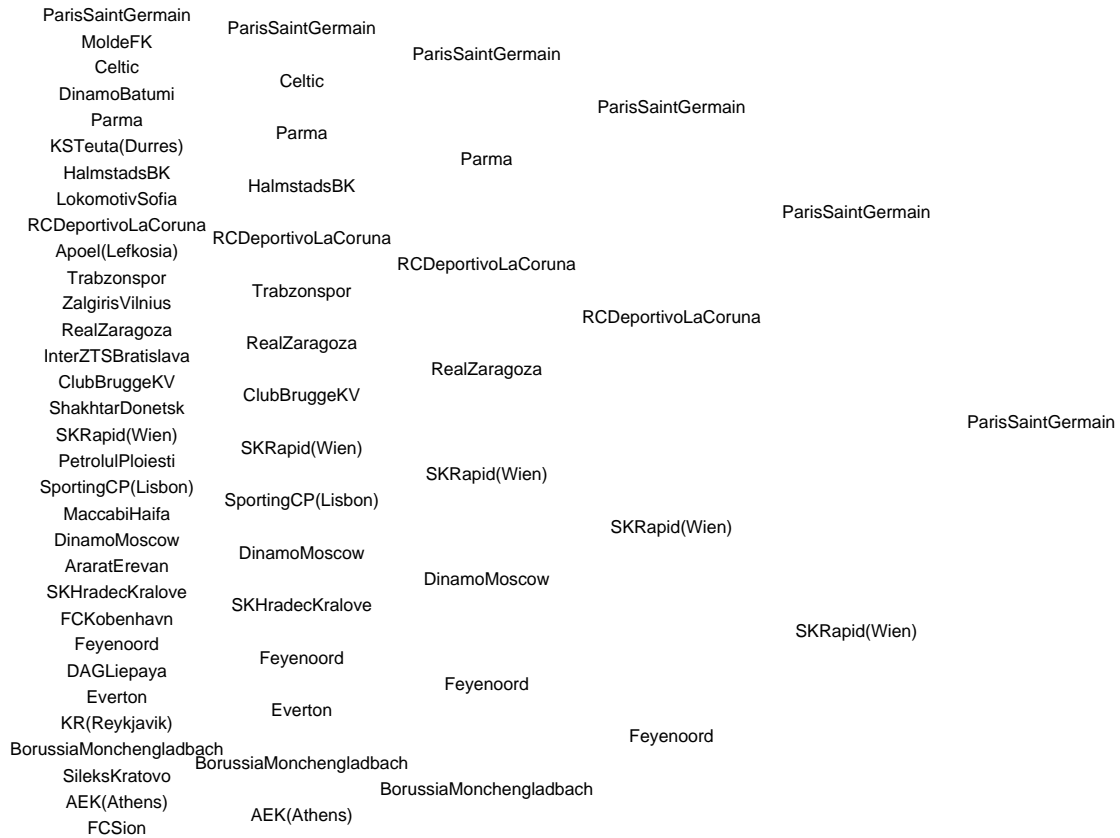


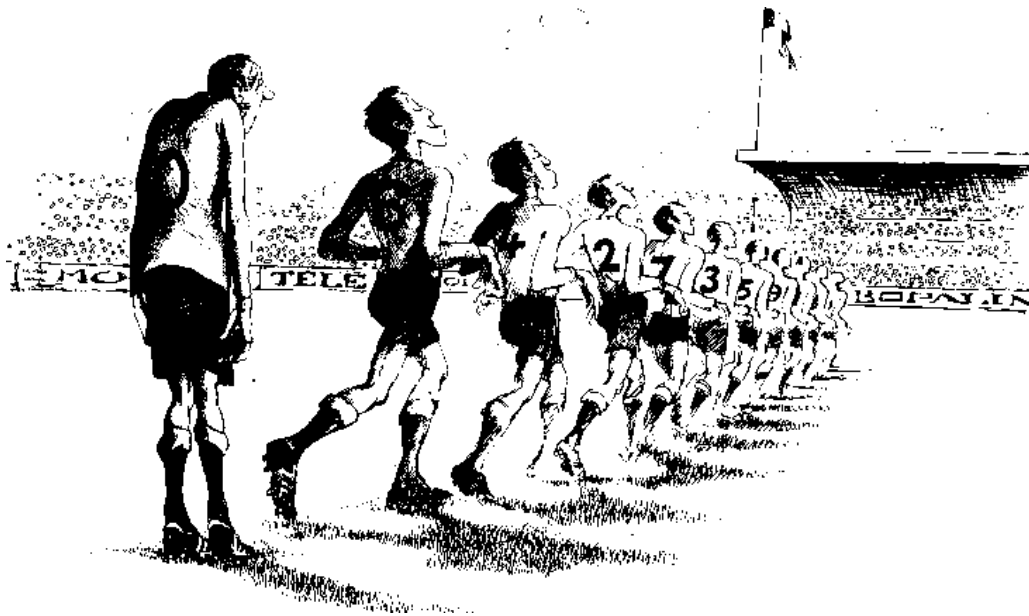
Figure 8 *Tournament tree of the Cup Winners Cup 1995/96.*

One remarks the surprising performance of SK Rapid (Wien) (Aut): Their chance to reach the quarter final was 16.7%, but they even reached the semi final where their chance was only 6.17%. Knowing this surprising fact the probabilities of the other teams remaining in the tournament will change. Therefore it is not only interesting to know the probability of each team winning in a chosen leg at the beginning of the tournament, but also predicting the probability of each team winning knowing the remaining teams. That is why we refined the program `prob.cte`. For example in the Cup Winners Cup 1995/96 the following teams remain after the quarter finals : Paris Saint Germain (Fra), RC Deportivo La Coruna (Esp) , SK Rapid (Wien) (Aut) and Feyenoord (Ned). We are able to calculate the new probabilities knowing this fact. It is important to notice that these new probabilities are calculated without care of the outcomes of the previous rounds.

A more advanced method will be discussed in section 3.3. The result of the function is shown by the following table:

	Semi Final	
ParisSaintGermain	72.59	47.47
RCDeportivoLaCoruna	27.41	11.80
SKRapid(Wien)	20.90	4.07
Feyenoord	79.10	36.66

We notice that this time the chance of Paris Saint Germain to win the tournament is about 50% - and they won it. And the chance of SK Rapid (Wien) (Aut) to defeat their opponent, Feyenoord (Ned), in the semi final is about 21%. At the beginning of the tournament the chance of SK Rapid (Wien) (Aut) to win the semi final was only about 2%. So this approach seems to be more realistic.



3.2.3 Number of Correct Predictions



Let us consider the following table:

Team	Probability of winning
A	X
B	$1 - X$

where $X = P(A, B)$. If $X > \frac{1}{2}$ the probable winner will be A and if the real winner is also A we will mark a point, if $X < \frac{1}{2}$ ($1 - X > \frac{1}{2}$) and the real winner is B we will mark a point as well and if $X = 1 - X = \frac{1}{2}$ we will mark half a point (this semi point is justified by the fact that the expectation of a point is $0 \cdot \frac{1}{2} + 1 \cdot \frac{1}{2} = \frac{1}{2}$).

The following example issues from the Cup Winners Cup 1992/93 and shows the probabilities of winning the quarter final for the teams remaining in the tournament.

	Quarter
Parma	43.47
SpartaPrahá	56.53
AtleticoMadrid	74.09
Olympiakos(Pireus)	25.91
RoyalAntwerp	31.26
SteauaBucharest	68.74
SpartakMoscow	60.36
Feyenoord	39.64

As we know that the real winners were Parma, Atletico Madrid, Royal Antwerp and Spartak Moscow, it is possible to calculate the number of correct predictions. That would be 2 because Atletico Madrid and Spartak Moscow are probable winners as well.

In applying this procedure to the entire tournament we will get a list of points whose sum represents the number of correct predictions. For this reason we have programmed a function titled `nb.pronos.ok`. In using this function for various data sets we obtained the following table:

YEAR	OK	GAMES	%OK
cwc.9293	21.5	31	69.35
cwc.9394	20	31	64.52
cwc.9495	20.5	31	66.13
cwc.9596	19	31	61.29
uc.9293	45	63	71.43
uc.9394	33.5	63	53.17
uc.9495	45.5	63	72.22
uc.9596	37.5	63	59.52

OK stands for the number of correct predictions, GAMES represents the number of games per tournament and %OK the percentage of games predicted correctly.



3.3 Variation of Team Strength

This time we assume that the team strength (the seeding coefficients) do not remain constant throughout the tournament - they are depending on the leg.

3.3.1 Model Chosen

The principal idea is to calculate for each leg a new coefficient for each team. The initial coefficients that are used to predict the issues of the first leg are the coefficients calculated in section 2.3.2. For the second round we have to calculate new coefficients. To do so we introduce two different methods (see sections 3.3.2 and 3.3.3). Once we have calculated these new coefficients we center and reduce them in order to obtain coefficients of equal mean and equal standard deviation as the initial ones.

Justification: Let x_1, \dots, x_n be the initial coefficients of mean \bar{x} and standard deviation s_x and let y_1, \dots, y_n be the new coefficients of mean \bar{y} and standard deviation s_y . We define the new coefficients as

$$y_t^* = \bar{x} + s_x \left(\frac{y_t - \bar{y}}{s_y} \right), \quad t = 1, \dots, n.$$

Hence $\bar{y}^* = \bar{x}$ and $s_{y^*} = s_x$. An example of this calculation is given by the following table:

Team	x_t	y_t	y_t^*
A	1.4	2.1	1.382
B	1.2	1.9	1.241
C	1	1.5	0.958
D	0.8	1.3	0.817
Mean	1.1	1.7	1.1
Standard Deviation	0.258	0.365	0.258

Therefore we are allowed to use the estimations made in section 3.2.1. Thus our model is given by equation (3):

$$P_k(i, j) = \frac{e^{\hat{\beta}(S_k(i) - S_k(j))}}{1 + e^{\hat{\beta}(S_k(i) - S_k(j))}} \quad (5)$$

where $k = 1, \dots, \log_2 n$ and $\hat{\beta} = 1.2610$.

Remark on the effect of the centering and the reduction of the coefficients: If a strong team wins against a weak team their coefficient will decrease, but if a weak team defeats a strong team their coefficient will increase. That seems to be logical.

3.3.2 Using Probable Opponents

To illustrate this first method, let us have a look at figure 9.

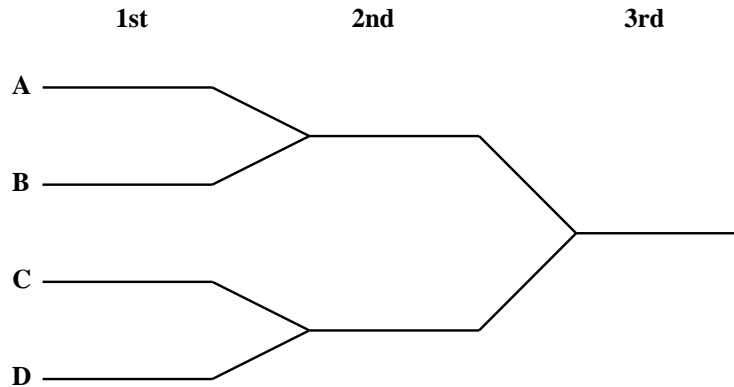


Figure 9 Example of the upper part of the tournament tree.

The initial coefficients are used to calculate the probability of each team winning against their potential opponent in the first round (for the first round only one potential opponent is possible). The new coefficients for the second round who will be used to calculate the probabilities of winning in the second round are defined in the following way, for example the new coefficient for a chosen team i would be:

$$C_{new,i} = C_{old,i} \left(1 + \frac{C_{old,O(i)} + \gamma}{C_{old,i} + \gamma} \right) \quad (6)$$

where γ is a constant ≥ 0 and $C_{old,O(i)}$ is the coefficient of the probable opponent $O(i)$ of team i in the previous round (if $i = A$ it would be B). Later on (see section 3.3.4) we will choose γ in order to maximize the number of correct predictions. It is important to notice that $C_{new,i}$ is $S_k(i)$ and $C_{old,i}$ is $S_{k-1}(i)$, $k = 1, \dots, \log_2 n$.

In realizing that some initial coefficients are equal to 0 (see section 2.3.2) we allow us to add 0.1 to all initial coefficients. Let us consider equation (6): If we will not do this the coefficients would remain 0 throughout the tournament. In adding 0.1 the expectation of the new initial coefficients increases and the variance remains the same. But as our model (5) is depending on a difference of coefficients this will not change our estimations.

In reusing equation (6) we are able to calculate the coefficients for the third round. The only problem consists in the knowledge of the probable opponent $O(i)$: Let i be A , so for the third round there are two potential opponents for A : C and D . But we have calculated $P_2(C, D)$ yet, so if this probability is $> \frac{1}{2}$ the probable opponent for A would be C , if not the probable opponent would be D (if this probability is equal to $\frac{1}{2}$ it does not matter which team we will take for further considerations because both teams have the same strength).

In generalizing this example we only have to keep in mind that if a team was not a

probable opponent for a previous round it will not be one for the next round. Therefore we have programmed a function titled `prob.non.cte`. An outcome of this function is shown by the following table where the Cup Winners Cup of the year 1993/94 has been considered.

	Round1	Round2	Quarter	Semi	Final
Arsenal	57.82	22.70	8.95	2.95	0.89
OB(Odense)	42.18	13.21	3.89	0.84	0.14
StandardCL(Liege)	62.84	42.79	21.46	9.19	3.61
CardiffCity	37.16	21.30	8.69	2.98	0.94
Torino	75.09	50.36	32.17	15.04	6.40
LillestromSK	24.91	10.71	4.54	1.29	0.33
Aberdeen	43.47	15.51	7.46	2.50	0.77
Valur(Reykjavik)	56.53	23.42	12.85	5.05	1.83
ParisSaintGermain	69.42	41.50	14.14	7.92	3.11
Apoel(Nicosia)	30.58	12.21	2.25	0.69	0.12
UniversitateaCraiova	65.26	33.72	10.62	5.63	2.09
HB(Torshavn)	34.74	12.57	2.29	0.70	0.12
RealMadrid	87.37	58.21	43.54	29.06	13.78
FCLugano	12.63	3.40	1.30	0.44	0.09
FCTirolInnsbruck	46.50	17.26	11.42	6.83	2.88
Ferencvarosi	53.50	21.13	14.43	8.88	3.86
Parma	81.64	49.27	23.79	14.19	8.69
DegerforsIF	18.36	5.42	1.07	0.27	0.08
MaccabiHaifa	22.08	5.63	1.10	0.28	0.08
TorpedoMoscow	77.92	39.67	18.19	10.50	6.28
Ajax	82.74	64.41	40.11	25.35	16.15
HajdukSplit	17.26	9.28	3.84	1.79	0.92
Besiktas	72.03	22.03	10.90	5.84	3.32
Kosice.1.FC	27.97	4.28	1.00	0.25	0.07
SLBenfica	70.52	52.49	38.36	18.76	11.50
GSKKatowice	29.48	18.25	11.62	4.75	2.62
CSKA(Sofia)	72.03	24.48	15.60	6.38	3.52
FCBalzers	27.97	4.77	1.73	0.31	0.08
BayerLeverkusen	56.96	20.65	5.62	1.50	0.61
FCBobyBrno	43.04	12.76	2.52	0.41	0.09
Panathinaikos	76.25	55.45	22.27	9.01	4.94
Shelbourne	23.75	11.13	2.28	0.40	0.10

The real outcomes are shown by figure 10.

cwc.9394

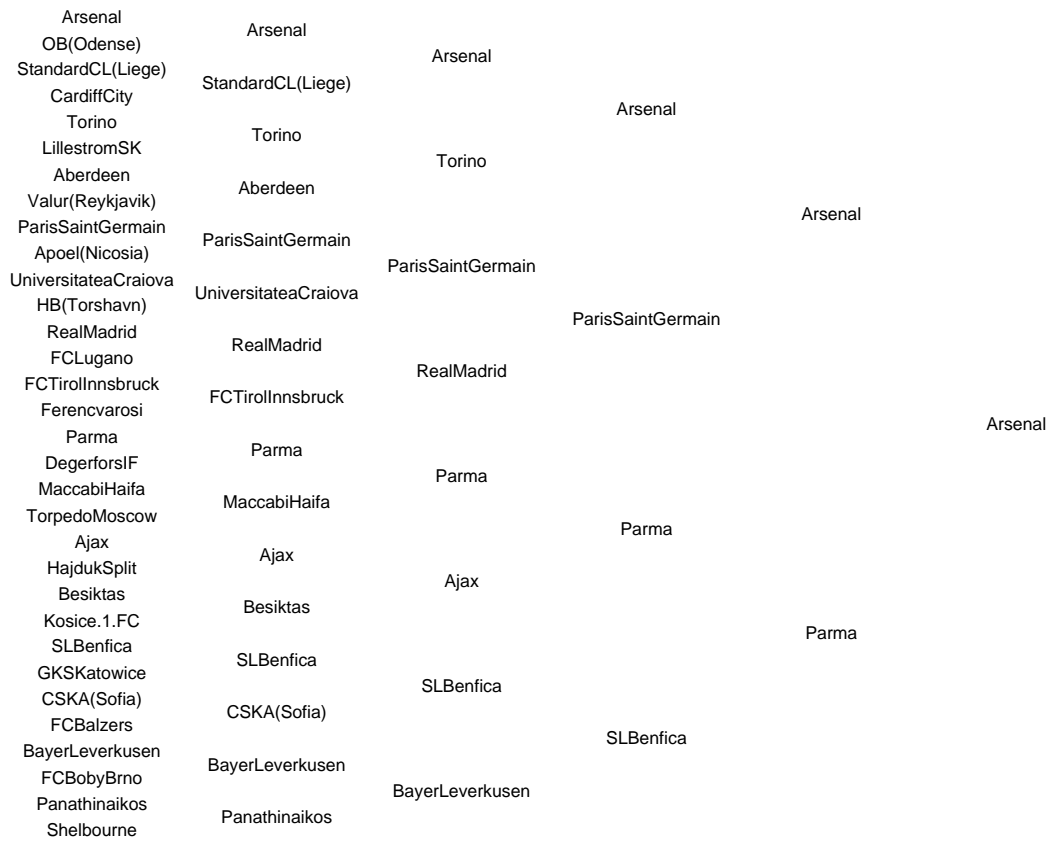


Figure 10 *Tournament tree of the Cup Winners Cup 1993/94.*

3.3.3 Using All Potential Opponents

The second method takes all potential opponents into consideration to calculate the new coefficients. This method seems to be more natural than the first one where we considered only the probable opponents. The new coefficient for a chosen team i is defined as:

$$C_{new,i} = C_{old,i} \sum_{j \in \mathcal{O}} (1 + r_j) P(j \text{ wins in the previous round}) \quad (7)$$

where γ is a constant ≥ 0 , $r_j = \frac{C_{old,j+\gamma}}{C_{old,i+\gamma}}$ and \mathcal{O} the set of all potential opponents for the previous round for the team i . One remarks that this new coefficient is a sort of weighted sum of old coefficients.

To illustrate this method let us have a look at the example shown by figure (9). For the second round the new coefficient is exactly the same as the one calculated for the second round in the previous section because there is only one potential opponent for the first round for each team. And for the third round the new coefficient of A would be

$$C_{new,A} = C_{old,A} \sum_{j \in \{C,D\}} (1 + r_j) P(j \text{ wins in the first round})$$

We have to notice that the procedure of calculating these new coefficients is the same as in the previous section. Hence we have reprogrammed the function `prob.non.cte`. An issue of this function is shown by the following table where the UEFA Cup of the year 1994/95 has been considered.

	Round1	Round2	Round3	Quarter	Semi	Final
Parma	73.00	47.03	34.40	19.37	12.02	6.50
Vitesse(Arnhem)	27.00	12.06	7.22	3.17	1.69	0.80
AIK(Stockholm)	57.82	25.47	16.84	8.37	4.82	2.45
SlaviaPrah	42.18	15.45	9.34	4.15	2.23	1.07
AthleticBilbao	50.00	17.92	3.66	0.75	0.21	0.05
AnorthosisofAmmocho	50.00	17.92	3.66	0.75	0.21	0.05
NewcastleUnited	27.34	13.41	2.59	0.49	0.12	0.02
RoyalAntwerp	72.66	50.75	22.29	10.39	5.75	2.82
OB(Odense)	42.18	17.53	4.95	1.87	0.85	0.35
Linfield	57.82	28.68	10.59	4.89	2.59	1.22
Kaiserslautern	53.06	29.23	11.30	5.39	2.91	1.40
IA(Akranes)	46.94	24.56	8.79	3.97	2.07	0.96
RealMadrid	63.33	40.23	27.80	16.57	10.39	5.66
SportingCP(Lisbon)	36.67	19.79	12.19	6.58	3.86	1.99
DinamoMoscow	77.92	36.41	23.36	13.05	7.84	4.13
RFC Seraing	22.08	3.58	1.02	0.25	0.07	0.02
BayerLeverkusen	53.15	34.87	18.71	10.21	4.60	2.17
PSV(Eindhoven)	46.85	29.65	15.20	8.01	3.47	1.59
KispestHonved	56.53	21.99	9.02	3.95	1.37	0.52
FCTwente(Enschede)	43.47	13.49	4.17	1.34	0.28	0.06

	Round1	Round2	Round3	Quarter	Semi	Final
GSKKatowice	65.26	30.80	16.44	8.55	3.65	1.65
Aris(Thessaloniki)	34.74	10.37	3.46	1.11	0.23	0.05
GirondinsBordeaux	60.36	37.54	22.15	12.50	5.84	2.83
LillestromSK	39.64	21.29	10.85	5.42	2.21	0.97
FCNantes	57.82	33.38	11.83	4.94	1.68	0.63
RotorVolograd	42.18	20.99	5.32	1.54	0.29	0.05
TekstilshchikKamyshin	50.00	22.82	5.85	1.72	0.33	0.06
BekescsabaiElore	50.00	22.82	5.85	1.72	0.33	0.06
FCSion	55.24	23.84	16.49	8.70	3.86	1.80
Apollon(Limassol)	44.76	16.96	11.06	5.42	2.22	0.97
OlympiqueMarseille	41.18	22.70	16.11	8.77	4.02	1.92
Olympiakos(Pireus)	58.82	36.51	27.49	16.10	7.95	4.01
Juventus	61.61	44.04	30.39	16.95	8.86	4.72
CSKA(Sofia)	38.39	24.48	15.33	7.72	3.69	1.85
CSMaritimo	50.00	15.74	7.90	3.11	1.18	0.50
FCAarau	50.00	15.74	7.90	3.11	1.18	0.50
AdmiraWacker	60.36	32.52	13.86	6.01	2.50	1.13
GornikZabrze	39.64	17.08	5.14	1.54	0.42	0.13
ASCannes	40.91	18.38	5.86	1.89	0.56	0.19
Fenerbahce	59.09	32.02	13.62	5.89	2.45	1.11
EintrachtFrankfurt	82.49	63.83	36.47	21.52	11.13	5.89
SCTOlimpija(Ljubljana)	17.51	8.46	2.08	0.55	0.10	0.02
RapidBucharest	50.00	13.86	3.68	1.08	0.25	0.06
RSCCharleroi	50.00	13.86	3.68	1.08	0.25	0.06
Napoli	57.82	22.87	11.14	5.43	2.34	1.09
SkontoRiga	42.18	13.21	5.20	2.06	0.71	0.28
Boavista	82.87	57.78	35.94	21.56	11.32	6.05
MyPa(Anjalankoski)	17.13	6.15	1.82	0.49	0.10	0.02
BorussiaDortmund	79.10	50.16	30.32	17.40	9.95	5.44
Motherwell	20.90	7.04	2.25	0.74	0.26	0.10
SlovanBratislava	50.00	21.40	10.45	5.07	2.56	1.27
FCKobenhavn	50.00	21.40	10.45	5.07	2.56	1.27
RCDeportivoLaCoruna	37.16	16.69	6.47	2.78	1.27	0.59
RosenborgBK	62.84	35.43	17.91	9.43	5.06	2.64
FCTirolInnsbruck	73.76	40.35	20.50	10.83	5.83	3.04
DinamoTbilisi	26.24	7.52	1.66	0.39	0.09	0.02
Lazio	65.26	45.23	19.68	9.32	4.74	2.38
DinamoMinsk	34.74	19.81	5.71	1.89	0.72	0.29
TrelleborgsFF	50.00	17.48	3.60	0.79	0.19	0.05
BlackburnRovers	50.00	17.48	3.60	0.79	0.19	0.05
Trabzonspor	63.25	37.75	26.93	15.07	8.63	4.73
DinamoBucharest	36.75	17.96	11.51	5.68	2.98	1.53
AstonVilla	34.74	12.05	6.90	2.98	1.42	0.67
Internazionale	65.26	32.25	22.08	11.77	6.52	3.48

The real outcomes are shown by figure 11.

uc.9495

Parma	Parma	Parma		
Vitesse(Arnhem)				
AIK(Stockholm)	AIK(Stockholm)			
SlaviaPraha			Parma	
AthleticBilbao	AthleticBilbao	AthleticBilbao		
AnorthosisofAmochostos				
NewcastleUnited	NewcastleUnited			
RoyalAntwerp			Parma	
OB(Odense)	OB(Odense)	OB(Odense)		
Linfield				
Kaiserslautern	Kaiserslautern		OB(Odense)	
IA(Akranes)				
RealMadrid	RealMadrid	RealMadrid		
SportingCP(Lisbon)				
DinamoMoscow	DinamoMoscow			Parma
RFC Seraing				
BayerLeverkusen	BayerLeverkusen	BayerLeverkusen		
PSV(Eindhoven)				
KispestHorved	KispestHorved		BayerLeverkusen	
FC Twente(Enschede)				
GKS Katowice		GKS Katowice		
GirondinsBordeaux	GirondinsBordeaux			
LillestromSK			BayerLeverkusen	
FCNantes	FCNantes	FCNantes		
RotorVolgograd				
TekstiltschikKamyshin	TekstiltschikKamyshin		FCNantes	
BekescsabaElore				
FC Sion	FC Sion	FC Sion		
Apollon(Limassol)				
OlympiqueMarseille	OlympiqueMarseille			
Olympiakos(Pireus)				Parma
Juventus	Juventus	Juventus		
CSKA(Sofia)	CSKA(Sofia)		Juventus	
CSMaritimo	CSMaritimo			
FCAarau				
AdmiraWacker	AdmiraWacker	AdmiraWacker		
GornikZabrze				
ASCannes	ASCannes		Juventus	
Fenerbahce				
EintrachtFrankfurt	EintrachtFrankfurt	EintrachtFrankfurt		
SCTOlimpija(Ljubljana)				
RapidBucharest	RapidBucharest		EintrachtFrankfurt	
RSCCharleroi				
Napoli	Napoli	Napoli		
SkontoRiga				
Boavista	Boavista			
MyPa(Anjalankoski)				Juventus
BorussiaDortmund	BorussiaDortmund	BorussiaDortmund		
Motherwell				
SlovanBratislava	SlovanBratislava			
FC København			BorussiaDortmund	
RCDeportivoLaCoruna	RCDeportivoLaCoruna	RCDeportivoLaCoruna		
RosenborgBK				
FC Tirol Innsbruck	FC Tirol Innsbruck			
Dinamo Tbilisi			BorussiaDortmund	
Lazio	Lazio	Lazio		
Dinamo Minsk				
Trelleborgs FF	Trelleborgs FF		Lazio	
Blackburn Rovers				
Trabzonspor	Trabzonspor	Trabzonspor		
Dinamo Bucharest				
Aston Villa	Aston Villa			
Internazionale				

Figure 11 Tournament tree of the UEFA Cup 1994/95.

3.3.4 Number of Correct Predictions

In regarding the two methods presented in the two preceding sections our goal in this section is to choose γ in order to maximize the number of correct predictions. The procedure of obtaining this number has been discussed in section 3.2.3. The calculation of the new coefficients for a certain round is based on the real outcomes of the previous round so there is only one potential opponent (the real one). The new coefficients will be calculated in using the following equation:

$$C_{new,i} = C_{old,i} \left(1 + \frac{C_{old,R(i)} + \gamma}{C_{old,i} + \gamma} \right)$$

where $R(i)$ is the real opponent of team i for the previous round. If a team is not in a particular round, *i.e.* it lost in one of the previous rounds, their coefficient has no reason to be changed. This remark is important because we will center and reduce the new coefficients in using all initial coefficients.

With $\gamma = 1$ we have obtained the following table:

YEAR	OK	GAMES	%OK

cwc.9293	21.5	31	69.35
cwc.9394	21	31	67.74
cwc.9495	20.5	31	66.13
cwc.9596	20	31	64.52
uc.9293	45	63	71.43
uc.9394	34	63	53.97
uc.9495	44	63	69.84
uc.9596	37	63	58.73

We have executed the function `nb.pronos.ok` for several values of γ , but how to know which γ is optimal? For this reason we have to define a criterion of comparison.

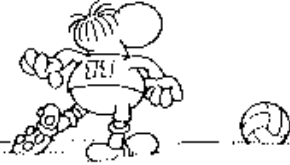
A general outcome of the function looks like this:

YEAR	OK	GAMES	%OK
1	o_1	g_1	p_1
2	o_2	g_2	p_2
\vdots	\vdots	\vdots	\vdots
8	o_8	g_8	p_8

Let W be $\sum_{i=1}^8 w_i p_i$, the weighted sum of the percentages, where w_i are the weights fulfilling $\sum_{i=1}^8 w_i = 1$ and $w_i \propto g_i$. Hence $w_i = g_i/g$ where $g = \sum_{i=1}^8 g_i$. By construction of the table $p_i = o_i/g_i$ therefore $W = (\sum_{i=1}^8 o_i)/g$.

So let us define our criterion of comparison:

Criterion: γ is optimal if $W = \frac{1}{g} \sum_{i=1}^8 o_i$ is maximal.



In calculating W for several values of γ we obtain the following table:

γ	0	0.2	0.4	0.6	0.8	1	2	3	4	5	15
W	58.51	62.5	63.29	64.36	64.36	64.62	64.62	64.62	64.36	64.09	64.09

One remarks that in taking for example $\gamma = 1$ we obtain the biggest W , *i.e.* the highest weighted sum of games predicted correctly. Therefore let 1 be the optimal γ to choose.

We also tried another model to calculate the new coefficients. This model is given by the following equation:

$$C_{new,i} = C_{old,i} \left(1 + \lambda \frac{C_{old,R(i)}}{C_{old,i}} \right)$$

We obtain the following table:

λ	0	0.2	0.4	0.6	0.8	1	2	3	4
W	64.5	64.5	64.22	63.56	63.56	58.51	58.9	59.3	60.1

One remarks that this time W does not exceed 64.5% as before. That is the reason why we do not take into further considerations this model.

In this section we chose γ in order to maximize the number of correct predictions. Our next goal will be to compare the methods presented in this paper (see section 4).

3.3.5 Evaluation of the Coefficients

General Remark

The results of this section have been achieved in considering several years. So let us have a look for example at the UEFA Cup 1995/96 in using the method that considers all potential opponents for a team. The other method furnishes the same results. In figure 12 the evaluation of the seeding coefficients throughout the whole tournament for four chosen teams is shown. Milan was the best seeded team at the beginning of the tournament, Olympiakos and Spartak Vladikavkaz were medium seeded teams and FC Lugano was among the worst seeded teams.

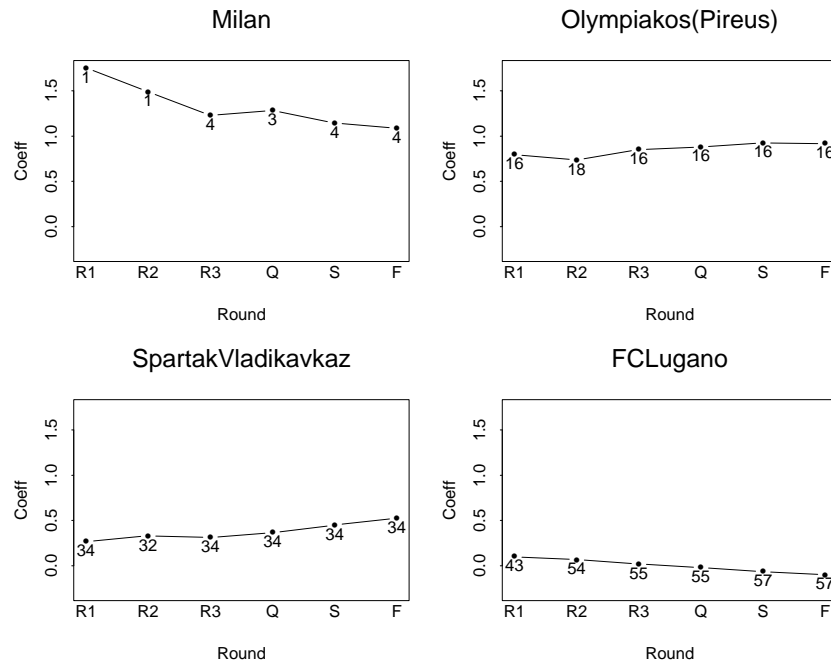


Figure 12 Evaluation of the seeding coefficients for four chosen teams during the UEFA Cup 1995/96. The number represents the seeding position of the team.

In general we noticed that the best teams, as Milan, will see their coefficient decrease, but they will still remain among the strongest teams. The medium teams, as Olympiakos and Spartak Vladikavkaz, will see their coefficients increase, and the worst teams will stay among the worst teams. The reason of this fact is that we centered and reduced the coefficients otherwise the better teams would always get higher coefficients and so become unbeatable.

In the appendix you will find the evaluation of the coefficients for all teams that have participated in the UEFA Cup 1995/96, in considering both methods.

Particular Examples

The two particular examples shown by figure 13 illustrate two things: First of all we remark that the two methods used are different, and in a second time we notice that in using the method considering only the probable opponent we have some strange evaluations: The coefficient of Milan decreases too much in the quarter final, they are losing six seed positions in falling from position 4 to position 10. This fact does not come up with the other method. The same can be said for Girondins Bordeaux in considering the coefficient for round 2. In recapitulating, the method using all potential opponents seems to be more natural.

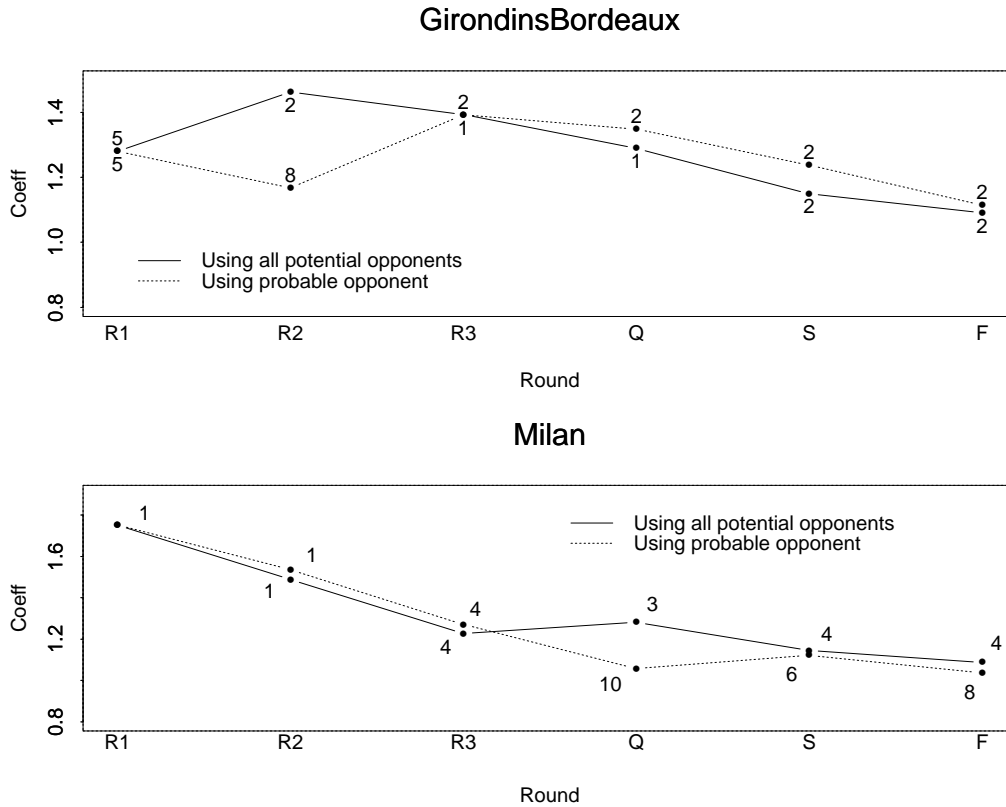
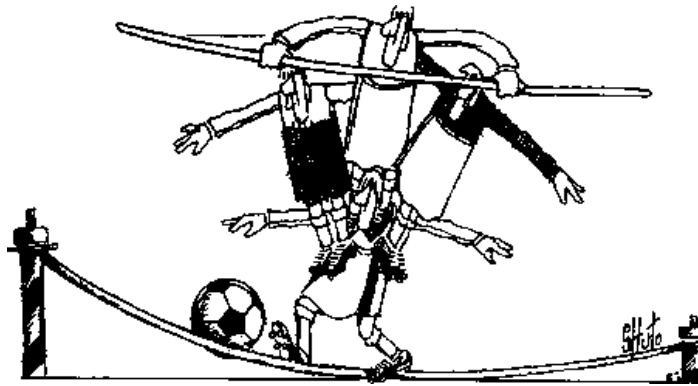


Figure 13 Evaluation of the seeding coefficients for Milan and Girondins Bordeaux during the UEFA Cup 1995/96.



4 Comparison of the Methods



In this section we want to compare the methods proposed in the previous sections:

- M1** The first method calculates the probabilities in using the constancy of the team strength.
- M2** The second one assumes the variation of the team strength in using only the probable opponents for a team.
- M3** The third one assumes the variation of the team strength in using all potential opponents for a team.

To compare the three methods **M1**, **M2** and **M3** we will take as example the quarter final of the UEFA Cup 1995/96. The table 1 shows the eight quarter finalists, the probability of each quarter finalist to reach the quarter final in using one of the three methods and the rank of this probability. This rank is defined as the order of the sorted probabilities of the team and of all his potential opponents for the previous rounds. Here is an example how this rank is calculated: Let us have a look at the following table who shows the probabilities to reach the quarter final for Bayern Muenchen and all his potential opponents for the previous rounds:

	Round3
BayernMuenchen	27.87
LokomotivMoscow	2.10
RaithRovers	2.80
IA(Akranes)	9.86
SLBenfica	46.03
LierseSK	1.40
RodaJC(Kerkrade)	2.58
SCTOlimpija(Ljubljana)	7.35

The rank of Bayern Muenchen would be 2 because SL Benfica had bigger chance to win the third round ($46.03\% > 27.87\%$), *i.e.* to reach the quarter final.

Team	M1		M2		M3	
	Prob.	Rank	Prob.	Rank	Prob.	Rank
Bayern Muenchen	23.99	2	26.33	2	27.87	2
Nottingham Forest	3.62	3	3.98	4	4.43	3
FC Barcelona	43.70	1	39.15	1	38.13	1
PSV (Eindhoven)	13.80	4	13.20	4	14.70	4
Girondins Bordeaux	29.18	1	25.04	2	29.21	1
Milan	67.68	1	60.62	1	56.71	1
Slavia Praha	22.88	2	25.06	3	25.17	2
Roma	10.19	3	10.16	4	11.40	3

Table 1 This table shows the eight quarter finalists, the probability of each one to reach the quarter final in using one of the three methods and the rank of this probability (UEFA Cup 1995/96).

For a balanced tournament all teams would have 12.5% chance to reach the quarter final because there are in all 7 potential opponents for each team in the previous rounds and in considering the team as well we have 8 teams sharing 100%. For the teams with probability smaller than 12.5% we remark that **M3** gives more chance to reach the quarter final than **M2**. One notices as well that Nottingham Forest, Girondins Bordeaux, Slavia Praha and Roma had more chance to reach the quarter final in using **M3** than in using **M2** because their ranks are smaller in using **M3**. Hence **M3** seems to be better than **M2**. On the other hand **M1** and **M3** deliver equal ranks.

Moreover, in comparing the number of correct predictions made in using one of the three methods we obtained the following table:

Year	% games predicted correctly	
	M1	M2 or M3
cwc.9293	69.35	69.35
cwc.9394	64.52	67.74
cwc.9495	66.13	66.13
cwc.9596	61.29	64.52
uc.9293	71.43	71.43
uc.9394	53.17	53.97
uc.9495	72.22	69.84
uc.9596	59.52	58.73

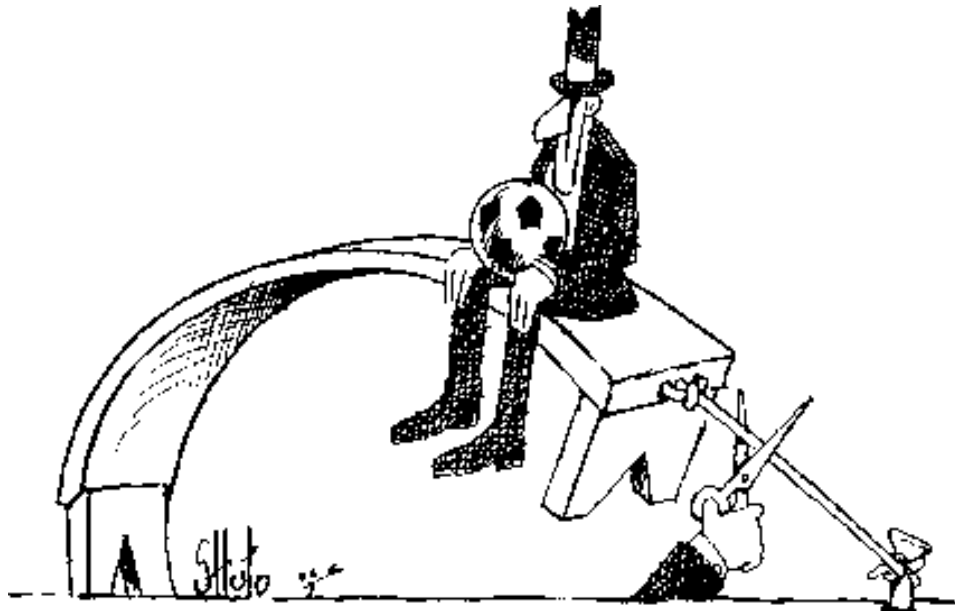
This table is a result from sections 3.2.3 and 3.3.4 and from the fact that for **M2** and **M3** we have the same number of correct predictions. Hence the feeling we had in the previous sections, *i.e.* that **M3** is the best method proposed, does not seem to be confirmed. **M3** does not predict more games correctly. Let us remind that for **M1** we obtained a weighted sum of games predicted correctly equal to 64.49% (see section 3.2.3) and for **M3** we obtained 64.62% (see section 3.3.4). At the end of section 3.3.4 we even tried another method to calculate the new coefficients where we obtained 64.5%.

Therefore we are able to conclude:

M1 which assumes the constancy of the team strength is the most appropriated method among the three.

This conclusion is based on the following points:

1. In assuming variation of the team strength we do not obtain better results. This statement has been verified in using 376 game outcomes, although the model was fitted on 442 game outcomes.
2. The practical application of **M1** is easier, faster in time and does not involve the search of an optimal γ .



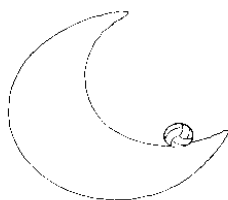
5 Conclusion

In this paper, the logistic regression model (3) using seed positions (conceived through a seeding coefficient) was applied to European soccer Cups tournament data in order to predict the probability of winning the tournament for each one of the participating teams, and the predicted probabilities of each team reaching a certain leg such as the quarter final. We proposed three different methods to do this: The first one assumed the constancy of the team strength and the two others assumed the variation of the team strength. In the previous section we compared the three methods and the conclusion was the following one: The best method is the one who assumes the constancy of the team strength. For every soccer fan this seems to be rather strange, but we obtained this fact in considering 376 game outcomes (!) of the European Cups from 1992 to 1996. So let us recapitulate this method which can be divided into two major parts:

1. Calculation of the weighted coefficients: One calculates a weighted mean of the ratio points achieved over games played for each of the three past years (see section 2.3.2).
2. Calculation of the probability of each team reaching a certain leg to obtain finally the probability of winning the tournament in using equations (1) and (2) (see section 2.4.2) with help of the logistic regression model (3).

Besides we remarked in section 3.2.3 that in using this method we had in average 64.49% of the games predicted correctly.

The attentive reader has certainly detected that we never took into consideration the scores. To calculate the new coefficient for a team we did not consider the goals for and against the teams in the tournament. This would surely be an interesting starting point to develop a new method for calculating new coefficients in hope to get a better method than the one who assumes the constancy of the team strength.



A Appendix

A.1 The Bradley-Terry Model

The Bradley-Terry model (Bradley and Terry 1952) is a model representing the results of experiments in which responses are pairwise rankings of treatments (so-called paired comparison experiments). Let us consider t treatments T_1, \dots, T_t in an experiment involving paired comparisons. We consider that these treatments have true ratings (or preferences) π_1, \dots, π_t . These parameters represent relative selection probabilities for the treatments, subject to the constraints $\pi_i \geq 0$ ($i = 1, \dots, t$) and $\sum_{i=1}^t \pi_i = 1$. Bradley and Terry (1952) defined the probability that treatment T_i is preferred over the treatment T_j in a single comparison in the following way:

$$P(T_i \rightsquigarrow T_j) = \frac{\pi_i}{\pi_i + \pi_j} \quad (8)$$

where $i \neq j$ and $i, j = 1, \dots, t$.

But what is the relationship between this model and our logistic model?

Let be $\pi_i = e^{\beta S_k(i)}$ and $\pi_j = e^{\beta S_k(j)}$ the ratings (or preferences) representing relative selection for the teams. Hence our logistic model (3) becomes:

$$\begin{aligned} P_k(i, j) &= \frac{e^{\alpha} \pi_i \pi_j^{-1}}{1 + e^{\alpha} \pi_i \pi_j^{-1}} \\ &= \frac{e^{\alpha} \pi_i}{\pi_j + e^{\alpha} \pi_i}. \end{aligned} \quad (9)$$

In section 3.2.1 we noticed that the regression rejects the necessity of the intercept α , hence equation (9) becomes:

$$P_k(i, j) = \frac{\pi_i}{\pi_j + \pi_i}$$

Therefore we can conclude that in rejecting the necessity of the intercept we obtain exactly the Bradley-Terry model (8) for a single comparison. The reject of $H_0 : \alpha = 0$ is also explicable by the following fact: α represents in a way the home advantage. But as for every round of the tournament two games are played because each team plays once at home and once on the road, hence the home advantage can be omitted.

A.2 Tournament Trees

A.2.1 Cup Winners Cup

cwc.8990



Figure 14 The Cup Winners Cup 1989/90.

cwc.9091

ManchesterUnited	ManchesterUnited	ManchesterUnited	
PecsiMunkasSC			
Wrexham	Wrexham		
LyngbyBK			ManchesterUnited
MontpellierHSC	MontpellierHSC		
PSV(Eindhoven)		MontpellierHSC	
SteauaBucharest	SteauaBucharest		
Glentoran			ManchesterUnited
LegiaWarsaw	LegiaWarsaw		
SwiftHesperange		LegiaWarsaw	
Aberdeen	Aberdeen		
NEASalamisofFamagusta			LegiaWarsaw
Sampdoria	Sampdoria		
Kaiserslautern		Sampdoria	
Olympiakos(Pireus)	Olympiakos(Pireus)		
KSFlamurtari(Vlore)			ManchesterUnited
FCBarcelona	FCBarcelona		
Trabzonspor		FCBarcelona	
Fram(Reykjavik)	Fram(Reykjavik)		
DjurgardensIF			FCBarcelona
DinamoKiev	DinamoKiev		
KuPS(Kuopio)		DinamoKiev	
DuklaPrague	DuklaPrague		
SliemaWanderers			FCBarcelona
Juventus	Juventus		
FCSliven		Juventus	
FKAustria(Wien)	FKAustria(Wien)		
PSVSchwerin			Juventus
RFCLiegeois	RFCLiegeois		
VikingFK(Stavanger)		RFCLiegeois	
EstrelaAmadora	EstrelaAmadora		
NeuchatelXamax			

Figure 15 *The Cup Winners Cup 1990/91.*

cwc.9192

WerderBremen	WerderBremen	WerderBremen	
FCBacau			
Ferencvarosi	Ferencvarosi	WerderBremen	
FCLevski(Sofia)			WerderBremen
Galatasaray	Galatasaray	Galatasaray	
EisenstadterFCStahl			
BanikOstrava	BanikOstrava		
OB(Odense)			WerderBremen
ClubBruggeKV	ClubBruggeKV	ClubBruggeKV	
Omonia(Nicosia)			
GSKKatowice	GSKKatowice		
Motherwell			ClubBruggeKV
AtleticoMadrid	AtleticoMadrid	AtleticoMadrid	
FyllingenIL			
ManchesterUnited	ManchesterUnited		
Athinaikos			WerderBremen
ASMonaco	ASMonaco	ASMonaco	
SwanseaCity			
IFKNorrkoping	IFKNorrkoping		
LaJeunesseDEsch			ASMonaco
Roma	Roma	Roma	
CSKAMoscow			
Ilves(Tampere)	Ilves(Tampere)		
Glenavon			ASMonaco
Feyenoord	Feyenoord	Feyenoord	
KSPartizani(Tirana)			
FCSion	FCSion		
Valur(Reykjavik)			Feyenoord
TottenhamHotspur	TottenhamHotspur	TottenhamHotspur	
HajdukSplit			
FCPorto	FCPorto		
Valletta			

Figure 16 The Cup Winners Cup 1991/92.

CWC.9293

Parma	Parma	Parma	
UjpestiTE	Boavista		
Boavista	Boavista		
Valur(Reykjavik)	SpartaPraha		Parma
SpartaPraha	SpartaPraha		
Airdrieonians	WerderBremen	SpartaPraha	
WerderBremen	WerderBremen		
Hannover96	AtleticoMadrid		Parma
AtleticoMadrid	AtleticoMadrid		
MariborBranik	Trabzonspor	AtleticoMadrid	
Trabzonspor	Trabzonspor		
TPS(Turku)			AtleticoMadrid
Olympiakos(Pireus)	Olympiakos(Pireus)		
ChernomoretsOdessa		Olympiakos(Pireus)	
ASMonaco	ASMonaco		
MiedzLegnica			Parma
RoyalAntwerp	RoyalAntwerp		
Glenavon			
AdmiraWacker	AdmiraWacker	RoyalAntwerp	
CardiffCity			
SteauaBucharest	SteauaBucharest		RoyalAntwerp
Bohemians		SteauaBucharest	
AGF(Arhus)	AGF(Arhus)		
AIK(Stockholm)			RoyalAntwerp
SpartakMoscow	SpartakMoscow		
AvenirBeggen			
Liverpool		SpartakMoscow	
Apollon(Limassol)	Liverpool		SpartakMoscow
Feyenoord	Feyenoord		
HapoelPetachTikva			
FCLucerne		Feyenoord	
FCLevski(Sofia)	FCLucerne		

Figure 17 *The Cup Winners Cup 1992/93.*

cwc.9394

Arsenal	Arsenal		
OB(Odense)			
StandardCL(Liege)	StandardCL(Liege)	Arsenal	
CardiffCity			Arsenal
Torino	Torino		
LillestromSK		Torino	
Aberdeen	Aberdeen		
Valur(Reykjavik)			Arsenal
ParisSaintGermain	ParisSaintGermain		
Apoel(Nicosia)		ParisSaintGermain	
UniversitateaCraiova	UniversitateaCraiova		
HB(Torshavn)			ParisSaintGermain
RealMadrid	RealMadrid		
FCLugano		RealMadrid	
FCTirolInnsbruck	FCTirolInnsbruck		
Ferencvarosi			Arsenal
Parma	Parma		
DegerforsIF		Parma	
MaccabiHaifa	MaccabiHaifa		
TorpedoMoscow			Parma
Ajax	Ajax		
HajdukSplit		Ajax	
Besiktas	Besiktas		
Kosice.1.FC			Parma
SLBenfica	SLBenfica		
GSKKatowice		SLBenfica	
CSKA(Sofia)	CSKA(Sofia)		
FCBalzers			SLBenfica
BayerLeverkusen	BayerLeverkusen		
FCBobyBrno		BayerLeverkusen	
Panathinaikos	Panathinaikos		
Shelbourne			

Figure 18 *The Cup Winners Cup 1993/94.*

CWC.9495

RealZaragoza				
GloriaBistrita	RealZaragoza			
TatranPresov		RealZaragoza		
DundeeUnited	TatranPresov			
Feyenoord			RealZaragoza	
ZalgrisVilnius	Feyenoord			
WerderBremen		Feyenoord		
MaccabiTelAviv	WerderBremen			RealZaragoza
Chelsea				
ViktoriaZizkov	Chelsea			
FKAustria(Wien)		Chelsea		
MariborBranik	FKAustria(Wien)			Chelsea
ClubBruggeKV				
SligoRovers	ClubBruggeKV			
Panathinaikos		ClubBruggeKV		
FCPirin(Blagoevgrad)	Panathinaikos			RealZaragoza
Arsenal				
Omonia(Lefkosia)	Arsenal			
BrondbyIF		Arsenal		
KSTirane	BrondbyIF			
AJAuxerre			Arsenal	
NKCroatia(Zagreb)	AJAuxerre			
Besiktas		AJAuxerre		
HJK(Helsinki)	Besiktas			Arsenal
Sampdoria				
FKBodoGlimt	Sampdoria			
GrasshopperClubZuerich		Sampdoria		
ChernomoretsOdessa	GrasshopperClubZuerich			Sampdoria
FCPorto				
LKSLodz	FCPorto			
Ferencvarosi		FCPorto		
CSKAMoscow	Ferencvarosi			

Figure 19 *The Cup Winners Cup 1994/95.*

cwc.9596

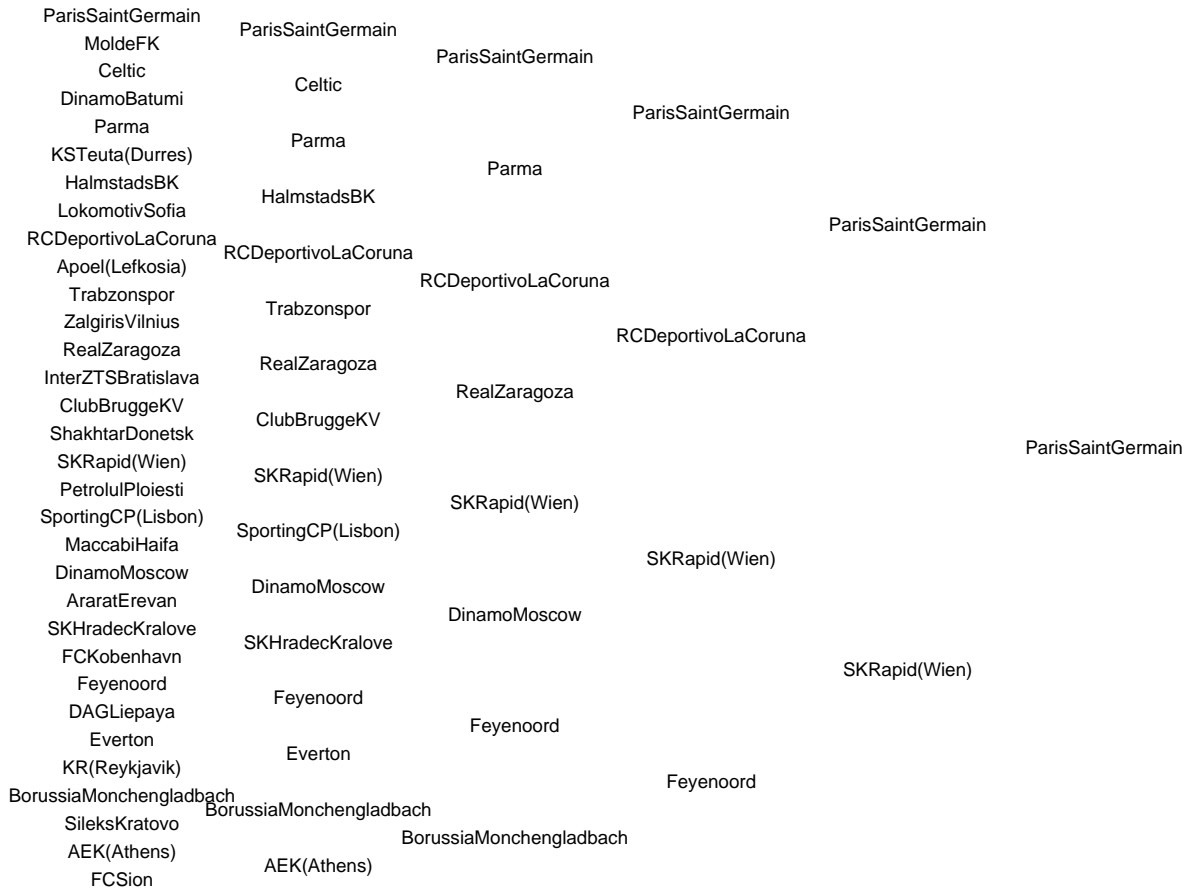


Figure 20 The Cup Winners Cup 1995/96.

A.2.2 UEFA Cup

uc.8990

Juventus	Juventus	Juventus		
GornikZabrze				
ParisSaintGermain	ParisSaintGermain			
FCKuusysi(Lahti)			Juventus	
FCKarlMarxStadt	FCKarlMarxStadt	FCKarlMarxStadt		
Boavista				
FC Sion	FC Sion			
Iraklis(Salonica)				
HamburgerSV	HamburgerSV	HamburgerSV		Juventus
OrgryteIS				
RealZaragoza	RealZaragoza	HamburgerSV		
Apollon(Limassol)			HamburgerSV	
FC Porto	FC Porto			
Flacarnioren		FC Porto		
Valencia	Valencia			
VictoriaBucharest				
Koeln	Koeln	Koeln		Juventus
TJPlastika(Nitra)				
SpartakMoscow	SpartakMoscow			
Atalanta			Koeln	
RedStar(Belgrade)	RedStar(Belgrade)	RedStar(Belgrade)		
Galatasaray				
ZalgrisVilnius	ZalgrisVilnius			
IFKGothenburg				
RoyalAntwerp	RoyalAntwerp	RoyalAntwerp		Koeln
FCLevski(Sofia)				
DundeeUnited	DundeeUnited			
GlenRoran			RoyalAntwerp	
VfBSuttgart	VfBSuttgart	VfBSuttgart		
Feyenoord				
ZenitLeningrad	ZenitLeningrad			
NaestvedIF				
ACFiorentina	ACFiorentina	ACFiorentina		Juventus
AtleticoMadrid				
FCSochauxMontbeliard	FCSochauxMontbeliard	ACFiorentina		
ASLJuvenesDEsch			ACFiorentina	
DinamoKiev	DinamoKiev			
MTK(Budapest)				
BanikOstrava	BanikOstrava	DinamoKiev		
FCHansaRostock				
AJAuxerre	AJAuxerre			ACFiorentina
KSApolonia(Fier)				
RoPS(Rovaniemi)	RoPS(Rovaniemi)	AJAuxerre		
GSKatowice				
Olympiakos(Pireus)	Olympiakos(Pireus)	Olympiakos(Pireus)		
FKRad(Belgrade)				
FirstViennaFC	FirstViennaFC			
Valletta				
WerderBremen	WerderBremen	WerderBremen		ACFiorentina
LillestromSK				
FKAustria(Wien)	FKAustria(Wien)	WerderBremen		
Ajax				
Napoli	Napoli	Napoli		
SportingCP(Lisbon)				
FCWettingen	FCWettingen			
Dundalk				
RFCLiegeois	RFCLiegeois	RFCLiegeois		WerderBremen
IA(Akranes)				
Hibernian	Hibernian			
VideotonWalthamSC				
SKRapid(Wien)	SKRapid(Wien)	SKRapid(Wien)		RFCLiegeois
Aberdeen				
ClubBruggeKV	ClubBruggeKV			
FCTwente(Enschede)				

Figure 21 The UEFA Cup 1989/90.

uc.9091

Internazionale SK Rapid(Wien)	Internazionale	Internazionale	Internazionale	
Aston Villa	Aston Villa			
Banik Ostrava				
FK Partizan(Belgrade)	FK Partizan(Belgrade)	FK Partizan(Belgrade)		
Hibernian				
Real Sociedad	Real Sociedad			
Lausanne Sports			Internazionale	
Atalanta	Atalanta	Atalanta		
NK Dinamo(Zagreb)	Fenerbahce			
Fenerbahce				
Vitoria SC(Guimaraes)	Koeln	Koeln	Atalanta	
Koeln				
IFK Norrkoping	Inter ZTS Bratislava			
Inter ZTS Bratislava				
Avenir Beggen	Sporting CP(Lisbon)			Internazionale
Sporting CP(Lisbon)		Sporting CP(Lisbon)		
KV Mechelen	Politehnica Timisoara			
Politehnica Timisoara				
Atletico Madrid	Vitesse(Arnheim)	Vitesse(Arnheim)		
Vitesse(Arnheim)				
Derry City	Dundee United			
Dundee United				
FH(Hafnarfjardar)	Bologna			
Bologna		Bologna	Sporting CP(Lisbon)	
Zagreb Lubin	Heart Of Midlothian			
Heart Of Midlothian				
Dnepr Dnepropetrovsk	Admira Wacker	Admira Wacker	Bologna	
Admira Wacker				
Vejle BK	FCLucerne			
FCLucerne				
MTK(Budapest)	Roma			Internazionale
Roma				
SL Benfica	Roma	Roma		
Valencia	Valencia			
Iraklis(Salonica)				
Girondins Bordeaux	Girondins Bordeaux			
Glenavon				
Magdeburg	Magdeburg			
ROPS(Rovaniemi)	RSC Anderlecht			
RSC Anderlecht		RSC Anderlecht		
Petrolul Ploiesti	Omonia(Nicosia)			
Omonia(Nicosia)				
FC Slavia(Sofia)	Borussia Dortmund			
Borussia Dortmund				
Chemnitz FC	Universitatea Craiova	Borussia Dortmund		
Universitatea Craiova				
KSPartizani(Tirana)	Brondby IF			Roma
Brondby IF				
Eintracht Frankfurt	Brondby IF	Brondby IF		
Ferencvárosi				
Royal Antwerp	Ferencvárosi			
Bayer Leverkusen				
Bayer Leverkusen	Bayer Leverkusen	Bayer Leverkusen		
FC Twente(Enschede)				
GKS Katowice	GKS Katowice			
TPS(Turku)				
Torpedo Moscow	Torpedo Moscow			Brondby IF
GAIS(Gothenburg)				
Sevilla	Sevilla	Torpedo Moscow		
PAOK(Salonica)				
AS Monaco	AS Monaco	AS Monaco	Torpedo Moscow	
Roda JC(Kerkrade)				
Chernomorets Odessa	Chernomorets Odessa			
Rosenborg BK				

Figure 22 The UEFA Cup 1990/91.

uc.9192

Ajax	Ajax	Ajax		
OrebroSK	FCRotWeissErfurt		Ajax	
FCRotWeissErfurt	FCRotWeissErfurt			
FCGroningen				
CAOsasuna	CAOsasuna		Ajax	
FCSlavia(Sofia)		CAOsasuna		
VfBSluttgart	VfBSluttgart			
PecsiMunkacsSC				Ajax
KAAGhent	KAAGhent			
LausanneSports		KAAGhent		
EintrachtFrankfurt	EintrachtFrankfurt			
CASporLuxembourg			KAAGhent	
DinamoMoscow	DinamoMoscow			
VacizzoMTE		DinamoMoscow		
ASCannes	ASCannes			Ajax
SCSalgueiros				
Genoa	Genoa			
RealOviedo		Genoa		
DinamoBucharest	DinamoBucharest			
SportingCP(Lisbon)			Genoa	
SteauaBucharest	SteauaBucharest			
AnorthosisofFamagusta		SteauaBucharest		
SportingGijon	SportingGijon			
FKPartizan(Belgrade)			Genoa	
Liverpool	Liverpool			
FCKuusysi(Lahti)		Liverpool		
AJAuxerre	AJAuxerre			
IkastFS			Liverpool	
FCTirolInnsbruck	FCTirolInnsbruck			
Tromsoll		FCTirolInnsbruck		
PAOK(Salonica)	PAOK(Salonica)			
KVMechelen				Ajax
Torino	Torino			
KR(Reykjavik)		Torino		
Boavista	Boavista			
Internazionale			Torino	
AEK(Athens)	AEK(Athens)			
SKVilaznia(Shkoder)		AEK(Athens)		
SpartakMoscow	SpartakMoscow			Torino
MP(Mikkeli)				
B1903(Copenhagen)	B1903(Copenhagen)			
Aberdeen		B1903(Copenhagen)		
BayernMunich	BayernMunich			
CorkCity			B1903(Copenhagen)	
Trabzonspor	Trabzonspor			
HASKGradjanski		Trabzonspor		
OlympiqueLyonnais	OlympiqueLyonnais			Torino
OstersIF				
RealMadrid	RealMadrid			
SlovanBratislava		RealMadrid		
FCUtrecht	FCUtrecht			
SKSturmGraz			RealMadrid	
NeuchatelXamax	NeuchatelXamax			
Floriana		NeuchatelXamax		
Celtic	Celtic			
KFCGerminal(Ekeren)			RealMadrid	
SKOlomoucSigma	SKOlomoucSigma			
Bangor		SKOlomoucSigma		
TorpedoMoscow	TorpedoMoscow			
HallescherFC			SKOlomoucSigma	
HamburgerSV	HamburgerSV			
GornikZabrze		HamburgerSV		
CSKA(Sofia)	CSKA(Sofia)			
Parma				

Figure 23 The UEFA Cup 1991/92.

uc.9293

Juventus	Juventus	Juventus	Juventus	
AnorthosisofAmmochostos	Panathinaikos			
Panathinaikos	SKOlomoucSigma	SKOlomoucSigma		
ElectroputereCraiova	Fenerbahce		Juventus	
SKOlomoucSigma	SLBenfica	SLBenfica		Juventus
UniversitateaCraiova	VacFCSamsung			
Fenerbahce	DinamoMoscow	DinamoMoscow		
FCBotev(Plovdiv)	Torino			Juventus
SLBenfica	ParisSaintGermain	ParisSaintGermain		
Belvedurtizola	Napoli			
VacFCSamsung	RSCAnderlecht	RSCAnderlecht		
FCGroningen	DinamoKiev			
DinamoMoscow	RealMadrid	RealMadrid		
RosenborgBK	RealMadrid			
Torino	TorpedoMoscow	TorpedoMoscow		
IFKNorrkoping	Vitesse(Arnhem)	Vitesse(Arnhem)		
ParisSaintGermain	KVMechelen			
PAOK(Thessaloniki)	BorussiaDortmund	BorussiaDortmund		Juventus
Napoli	Celtic			
Valencia	RealZaragoza	RealZaragoza		
RSCAnderlecht	BKFFrem(Kobenhavn)			
Hibernian	Roma	Roma		
DinamoKiev	GrasshopperClubZuerich			
SKRapid(Wien)	Galatasaray	Galatasaray		
RealMadrid	EintrachtFrankfurt			
PolitehnicaTimisoara	WidzewLodz			
TorpedoMoscow	AJAuxerre	AJAuxerre		
ManchesterUnited	FCLokomotiv(Plovdiv)			
Vitesse(Arnhem)	FCCKobenhavn	AJAuxerre		
DerryCity	MP(Mikkeli)			
KVMechelen	StandardCL(Liege)	StandardCL(Liege)		
OrebroSK	Portadown			
BorussiaDortmund	HeartOfMidlothian			
Floriana	SlaviaPraha			
Celtic	Ajax	Ajax		
Koeln	SVCasinoSalzburg			
RealZaragoza	VitoriaSC(Guimaraes)			
SMCaen	RealSociedad			
BKFFrem(Kobenhavn)	Kaiserslautern	Kaiserslautern		
NeuchatelXamax	Framt(Reykjavik)			
Roma	SheffieldWednesday			
FCWackerInnsbruck	CASporalLuxembourg			
GrasshopperClubZuerich				
SportingCP(Lisbon)				
Galatasaray				
GKSKatowice				
EintrachtFrankfurt				
WidzewLodz				
AJAuxerre				
FCLokomotiv(Plovdiv)				
FCCKobenhavn				
MP(Mikkeli)				
StandardCL(Liege)				
Portadown				
HeartOfMidlothian				
SlaviaPraha				
Ajax				
SVCasinoSalzburg				
VitoriaSC(Guimaraes)				
RealSociedad				
Kaiserslautern				
Framt(Reykjavik)				
SheffieldWednesday				
CASporalLuxembourg				

Figure 24 The UEFA Cup 1992/93.

uc.9394

Internazionale	Internazionale	Internazionale	Internazionale	
RapidBucharest	Apollon(Limassol)			
Apollon(Limassol)	NorwichCity	NorwichCity		
VacFCSamsung	BayernMuenchen			
NorwichCity	BorussiaDortmund	BorussiaDortmund		Internazionale
Vitesse(Arnhem)	MariborBranik			
BayernMuenchen	BrondbyIF	BrondbyIF		
FC Twente(Enschede)	FCKuusysi(Lahti)			
BorussiaDortmund	Cagliari	Cagliari		Internazionale
SpartakVladikavkaz	Trabzonspor			
MariborBranik	KVMechelen	KVMechelen		
GloriaBistrita	MTK(Budapest)			
BrondbyIF	Juventus	Juventus		
DundeeUnited	KongsvingerIL			
FCKuusysi(Lahti)	CDTenerife	CDTenerife		
KSVWaregem	Olympiakos(Pireus)			
Cagliari	SVCasinoSalzburg	SVCasinoSalzburg		Internazionale
DinamoBucharest	RoyalAntwerp			
Trabzonspor	SportingCP(Lisbon)	SportingCP(Lisbon)		
Valletta	Celtic			
KVMechelen	EintrachtFrankfurt	EintrachtFrankfurt		
IFKNorrkoping	DneprDnepropetrovsk			
MTK(Budapest)	RCDeportivoLaCoruna	RCDeportivoLaCoruna		
KR(Reykjavik)	AstonVilla			
Juventus	KarlsruherSC	KarlsruherSC		
LokomotivMoscow	Valencia			
KongsvingerIL	GirondinsBordeaux	GirondinsBordeaux		
OstersIF	ServetteFCGeneva			
CDTenerife	Boavista	Boavista		
AJAuxerre	Lazio			
Olympiakos(Pireus)	OFI(Crete)	OFI(Crete)		
FCBotev(Plovdiv)	AtleticoMadrid			
SVCasinoSalzburg				
DACDunajskaStreda				
RoyalAntwerp				
CSMaritimo				
SportingCP(Lisbon)				
Kocaelispor				
Celtic				
BSCYoungBoys				
EintrachtFrankfurt				
DinamoMoscow				
DneprDnepropetrovsk				
AdmiraWacker				
RCDeportivoLaCoruna				
AaB(Aalborg)				
AstonVilla				
SlovanBratislava				
KarlsruherSC				
PSV(Eindhoven)				
Valencia				
FCNantes				
GirondinsBordeaux				
Bohemians				
ServetteFCGeneva				
Crusaders				
Boavista				
USLuxembourg				
Lazio				
FC Lokomotiv(Plovdiv)				
OFI(Crete)				
SlaviaPraha				
AtleticoMadrid				
HeartOfMidlothian				

Figure 25 The UEFA Cup 1993/94.

uc.9495

Parma	Parma	Parma		
Vitesse(Arnhem)	AIK(Stockholm)		Parma	
AIK(Stockholm)				
SlaviaPraha	AthleticBilbao	AthleticBilbao		
AthleticBilbao	NewcastleUnited			
AnorthosisofAmochostos	OB(Odense)	OB(Odense)	Parma	
NewcastleUnited	Kaiserslautern			
RoyalAntwerp	RealMadrid	RealMadrid	OB(Odense)	
OB(Odense)	DinamoMoscow			
Linfield	BayerLeverkusen	BayerLeverkusen		Parma
Kaiserslautern	KispestHonved			
IA(Akranes)	GKSKatowice	GKSKatowice	BayerLeverkusen	
RealMadrid	GirondinsBordeaux	GirondinsBordeaux		
SportingCP(Lisbon)	FCNantes	FCNantes	BayerLeverkusen	
DinamoMoscow	RotorVolgograd			
RFC Seraing	TekstilshchikKamyshin	TekstilshchikKamyshin		
BayerLeverkusen	FC Sion	FC Sion	FCNantes	
PSV(Eindhoven)	OlympiqueMarseille			
KispestHonved	Juventus	Juventus		Parma
FCTwente(Enschede)	CSMaritimo	CSMaritimo	Juventus	
GKSKatowice	AdmiraWacker	AdmiraWacker		
Aris(Thessaloniki)	ASCannes		Juventus	
GirondinsBordeaux	EintrachtFrankfurt	EintrachtFrankfurt		
LillestromSK	RapidBucharest	EintrachtFrankfurt	EintrachtFrankfurt	
FCNantes	Napoli	Napoli		
RotorVolgograd	Boavista			
TekstilshchikKamyshin	BorussiaDortmund	BorussiaDortmund		Juventus
BekescsabaEllore	SlovanBratislava	SlovanBratislava	BorussiaDortmund	
FC Sion	RCDDeportivoLaCoruna	RCDDeportivoLaCoruna		
Apollon(Limassol)	RosenborgBK			
OlympiqueMarseille	FCTirolInnsbruck	FCTirolInnsbruck		
Olympiakos(Pireus)	Lazio	Lazio	BorussiaDortmund	
Juventus	DinamoMinsk			
CSKA(Sofia)	TrelleborgsFF			
CSMaritimo	Trabzonspor	Trabzonspor		
FC Aarau	AstonVilla			
AdmiraWacker				
GornikZabrze				
ASCannes				
Fenerbahce				
EintrachtFrankfurt				
SCTOlimpija(Ljubljana)				
RapidBucharest				
RSCCharleroi				
Napoli				
SkontoRiga				
Boavista				
MyPa(Anjalankoski)				
BorussiaDortmund				
Motherwell				
SlovanBratislava				
FC Kobenhavn				
RCDDeportivoLaCoruna				
RosenborgBK				
FCTirolInnsbruck				
DinamoTbilisi				
Lazio				
DinamoMinsk				
TrelleborgsFF				
BlackburnRovers				
Trabzonspor				
DinamoBucharest				
AstonVilla				
Internazionale				

Figure 26 The UEFA Cup 1994/95.

uc.9596

BayernMuenchen	BayernMuenchen	BayernMuenchen		
LokomotivMoscow			BayernMuenchen	
RaithRovers	RaithRovers			
IA(Akranes)				
SLBenfica	SLBenfica			
LierseSK		SLBenfica		
RodaJC(Kerkrade)	RodaJC(Kerkrade)			
SCTOlimpija(Ljubljana)			BayernMuenchen	
NottinghamForest	NottinghamForest			
MalmöFF		NottinghamForest		
AJAuxerre	AJAuxerre			
VikingFK(Stavanger)			NottinghamForest	
OlympiqueLyonnais	OlympiqueLyonnais			
SCFarense		OlympiqueLyonnais		
Lazio	Lazio			
Omonia(Lefkosia)				BayernMuenchen
FCBarcelona	FCBarcelona			
HapoelBeerSheva		FCBarcelona		
VitoriaSC(Guimaraes)	VitoriaSC(Guimaraes)			
StandardCL(Liege)			FCBarcelona	
Sevilla	Sevilla			
FCBotev(Plovdiv)		Sevilla		
Olympiakos(Pireus)	Olympiakos(Pireus)			
MariborBranik			FCBarcelona	
PSV(Eindhoven)	PSV(Eindhoven)			
MyPa(Anjalankoski)		PSV(Eindhoven)		
LeedsUnited	LeedsUnited		PSV(Eindhoven)	
ASMonaco				
WerderBremen	WerderBremen			
Glenavon		WerderBremen		
DinamoMinsk	DinamoMinsk			
FKAustria(Wien)				BayernMuenchen
GirondinsBordeaux	GirondinsBordeaux			
VardarSkopje		GirondinsBordeaux		
RotorVolgograd	RotorVolgograd			
ManchesterUnited			GirondinsBordeaux	
RealBetisBalompie	RealBetisBalompie			
Fenerbahce		RealBetisBalompie		
Kaiserslautern	Kaiserslautern			
SlovanBratislava			GirondinsBordeaux	
Milan	Milan			
ZagrebLubin		Milan		
RCStrasbourg	RCStrasbourg			
UjpestiTE			Milan	
SpartaPrah	SpartaPrah			
SilkeborgIF		SpartaPrah		
ZimbruChisinau	ZimbruChisinau			
RAFYelgava			GirondinsBordeaux	
SlaviaPrah	SlaviaPrah			
SCFreiburg		SlaviaPrah		
FCLugano	FCLugano			
Internazionale			SlaviaPrah	
RCLens	RCLens			
AvenirBeggen		RCLens		
ChernomoretsOdessa	ChernomoretsOdessa			
WidzewLodz			SlaviaPrah	
Roma	Roma			
NeuchatelXamax		Roma		
SCEndrachtAalst	SCEndrachtAalst			
FCLevski(Sofia)			Roma	
BrondbyIF	BrondbyIF			
LillestromSK		BrondbyIF		
Liverpool	Liverpool			
SpartakVladikavkaz				

Figure 27 The UEFA Cup 1995/96.

A.3 Evaluation of the Coefficients

The UEFA Cup 1995/96 has been considered.

A.3.1 Using Probable Opponents

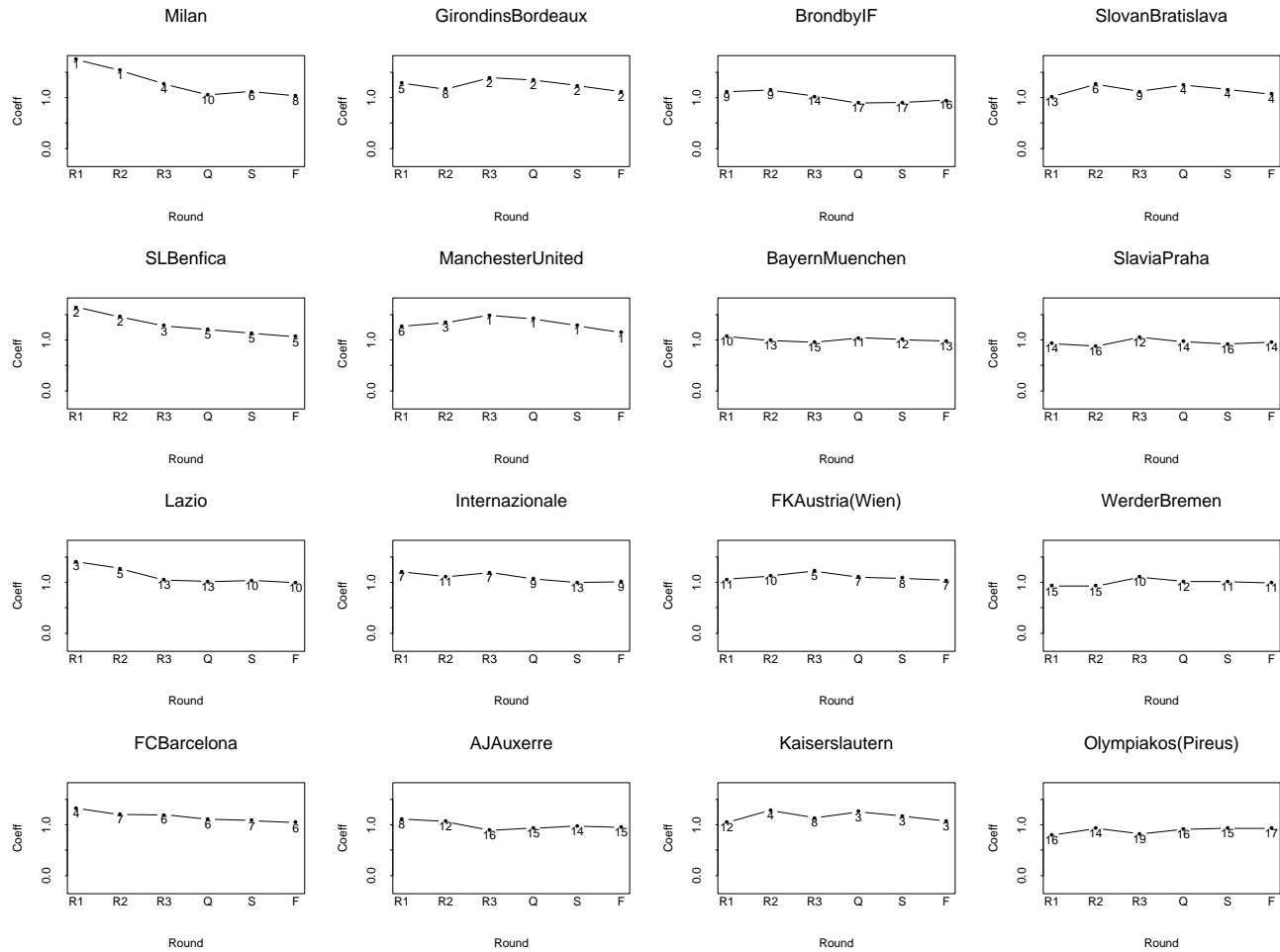


Figure 28 Using probable opponents (UEFA Cup 1995/96).

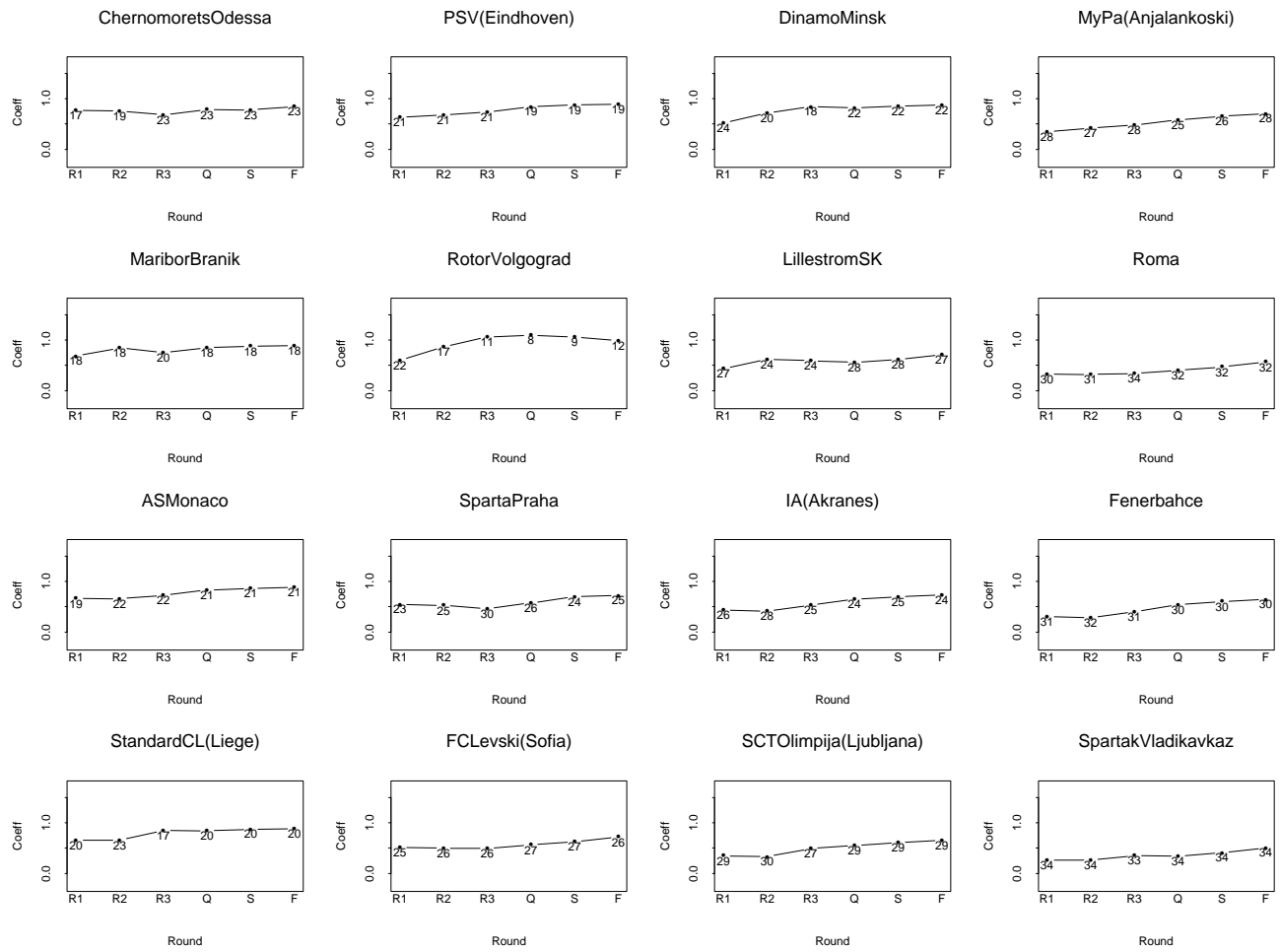


Figure 29 Using probable opponents (UEFA Cup 1995/96).

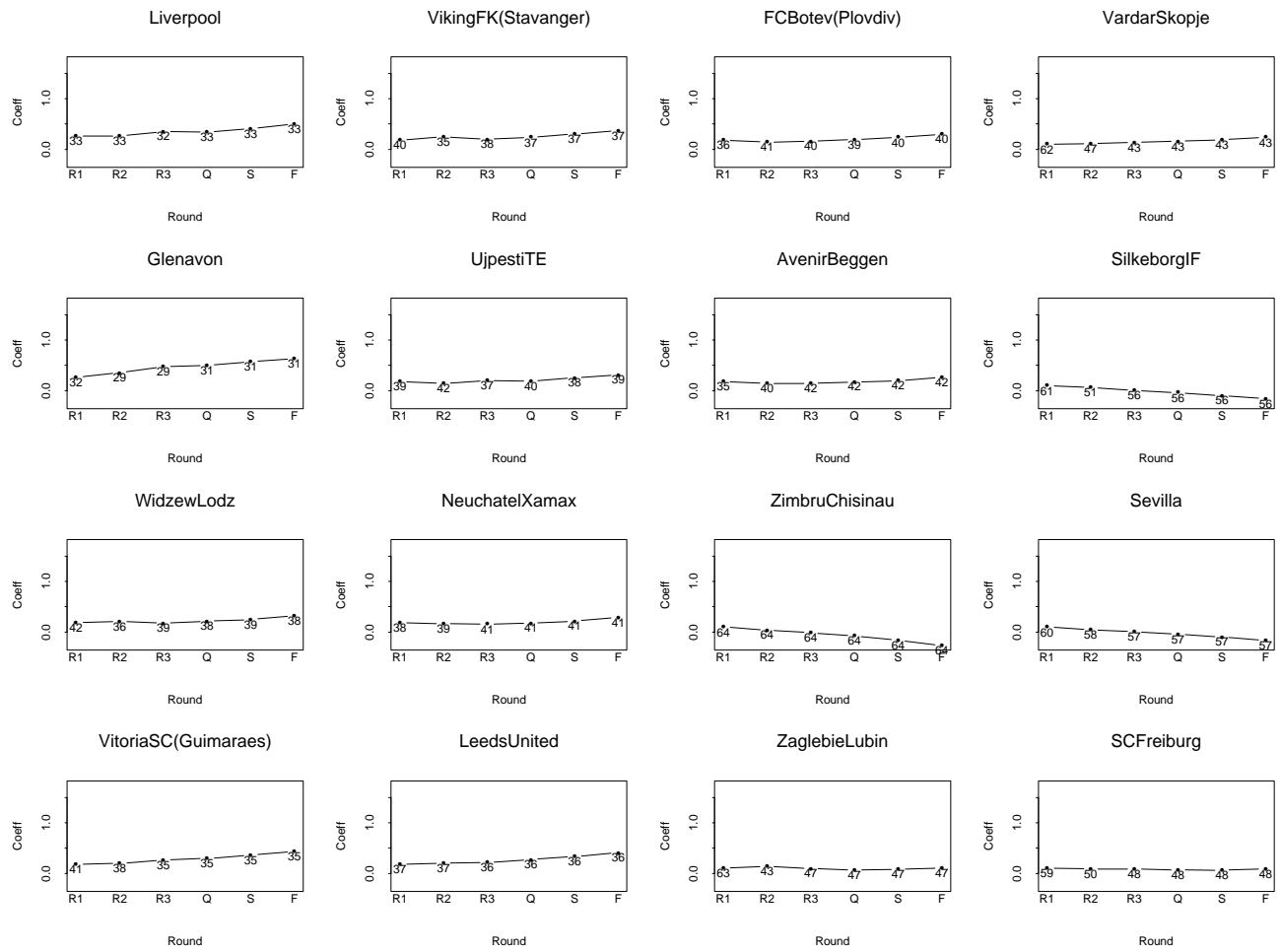


Figure 30 Using probable opponents (UEFA Cup 1995/96).

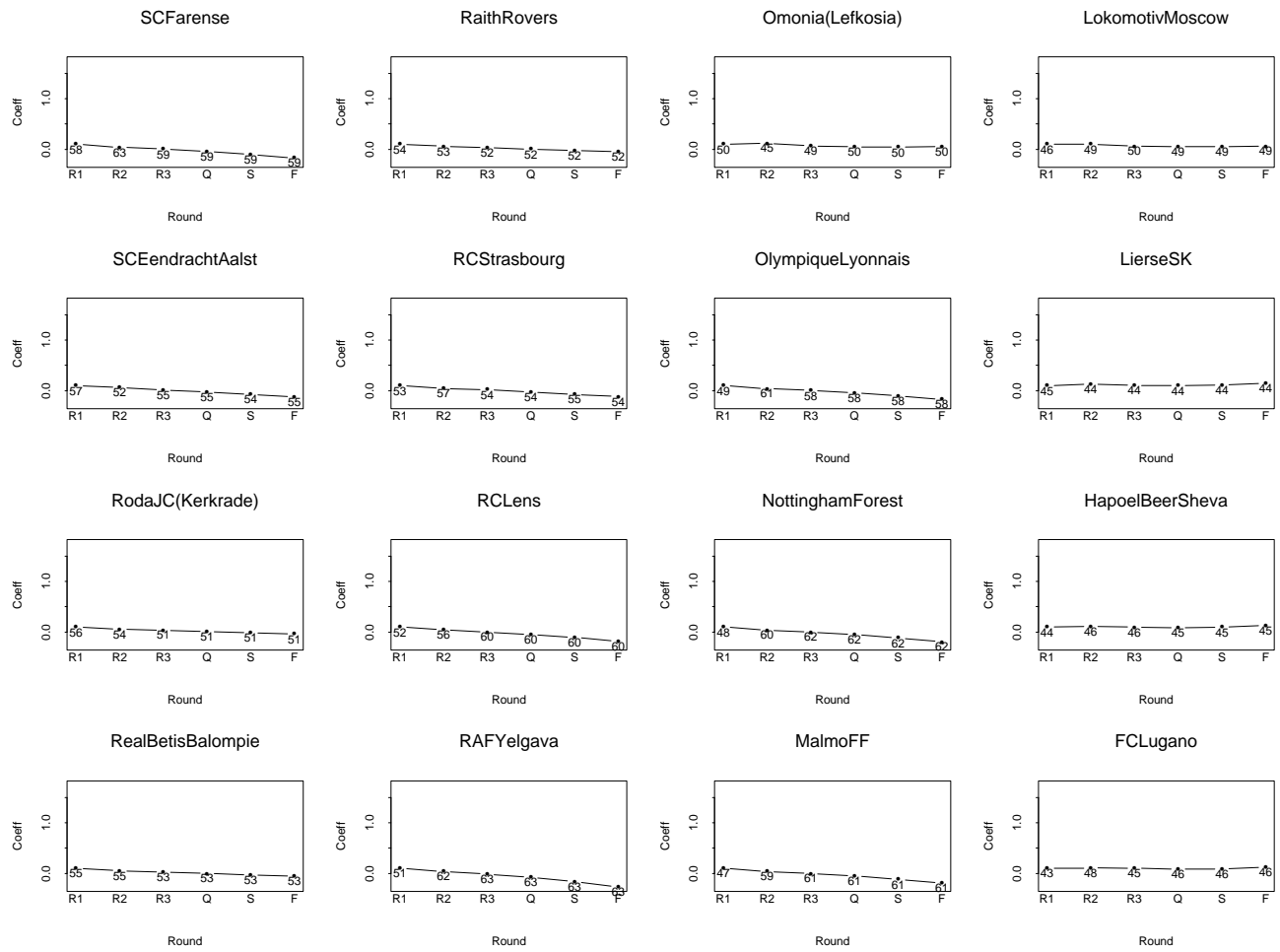


Figure 31 Using probable opponents (UEFA Cup 1995/96).

A.3.2 Using All Potential Opponents

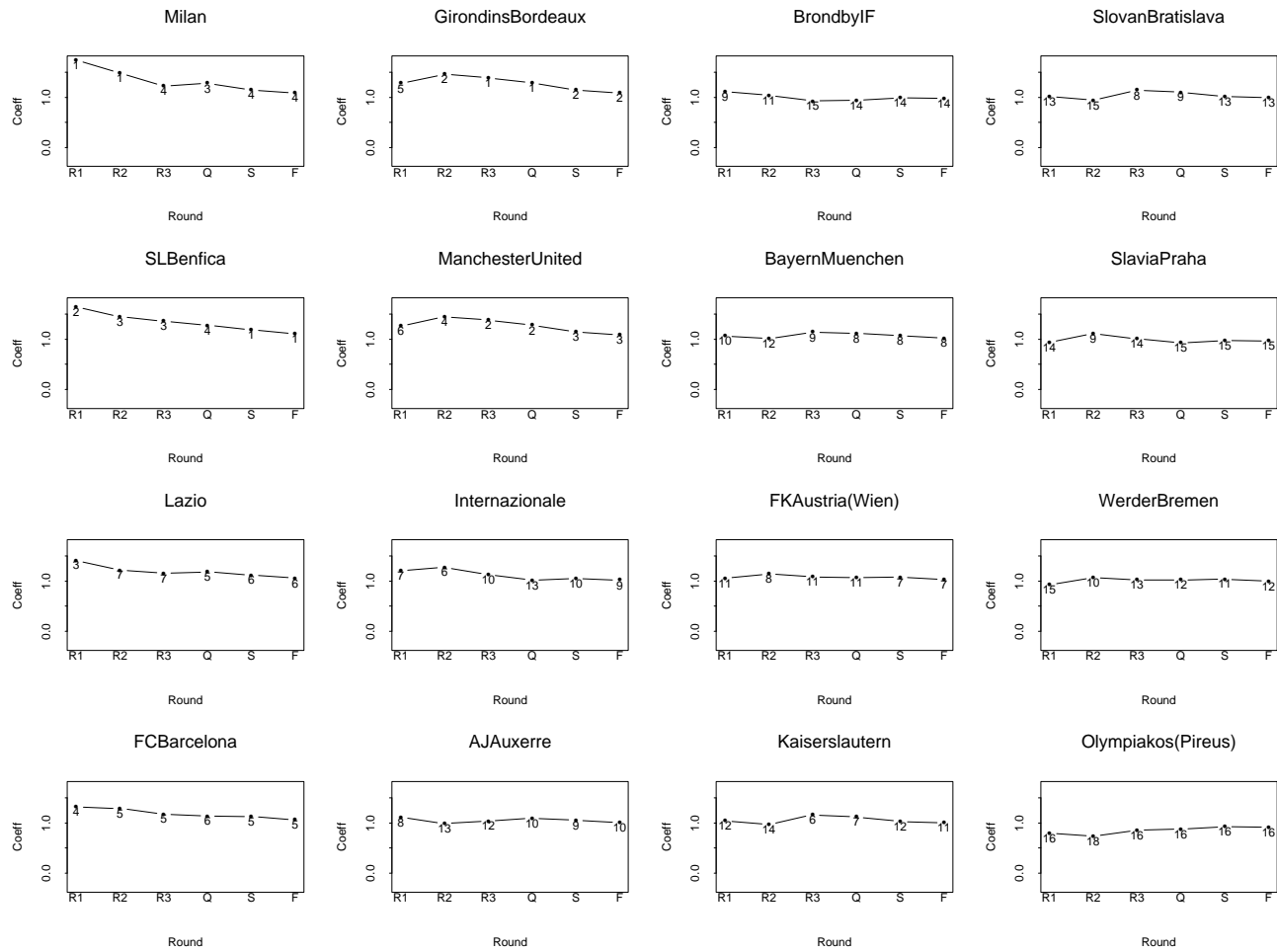


Figure 32 Using all potential opponents (UEFA Cup 1995/96).

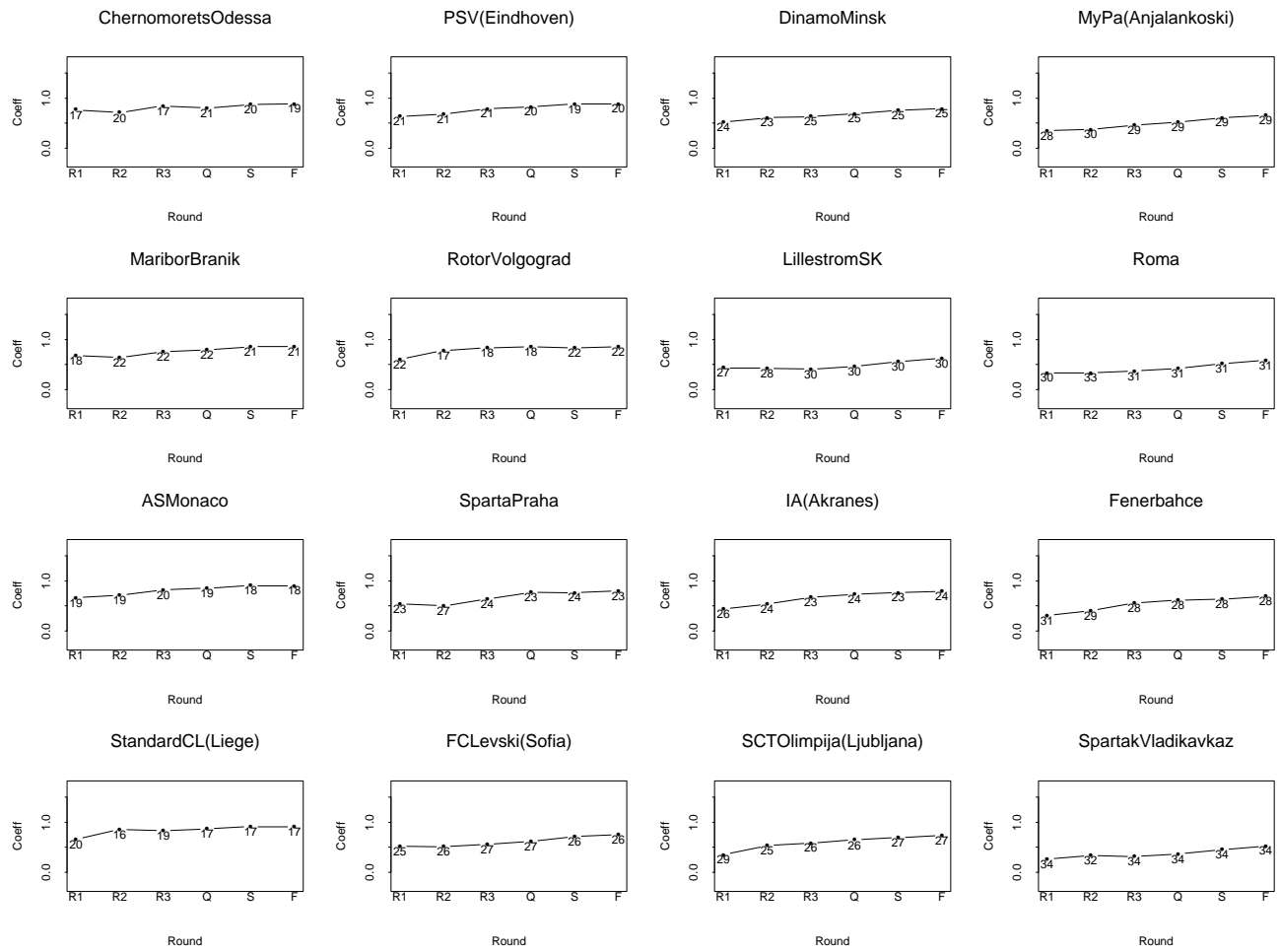


Figure 33 Using all potential opponents (UEFA Cup 1995/96).

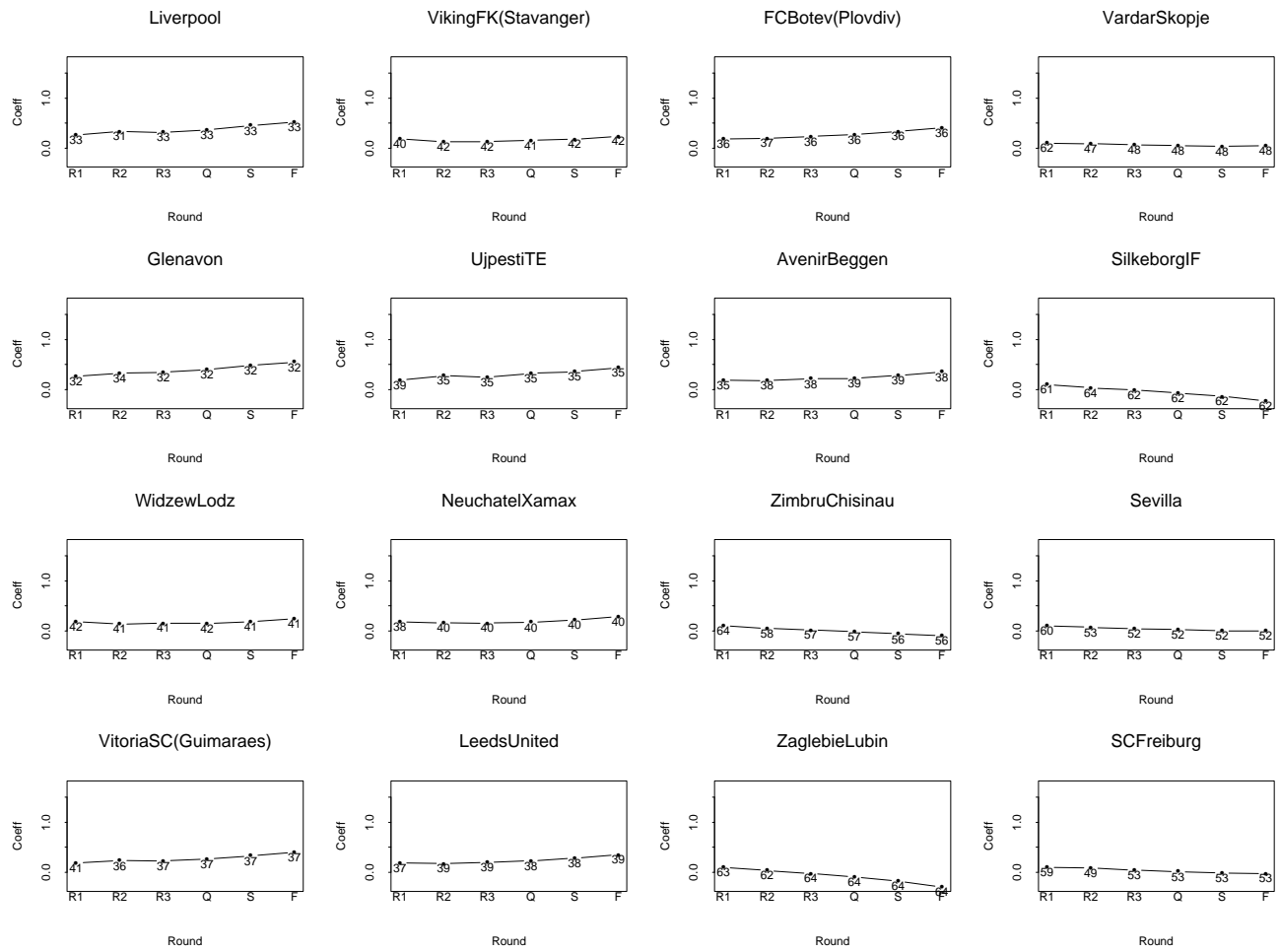


Figure 34 Using all potential opponents (UEFA Cup 1995/96).

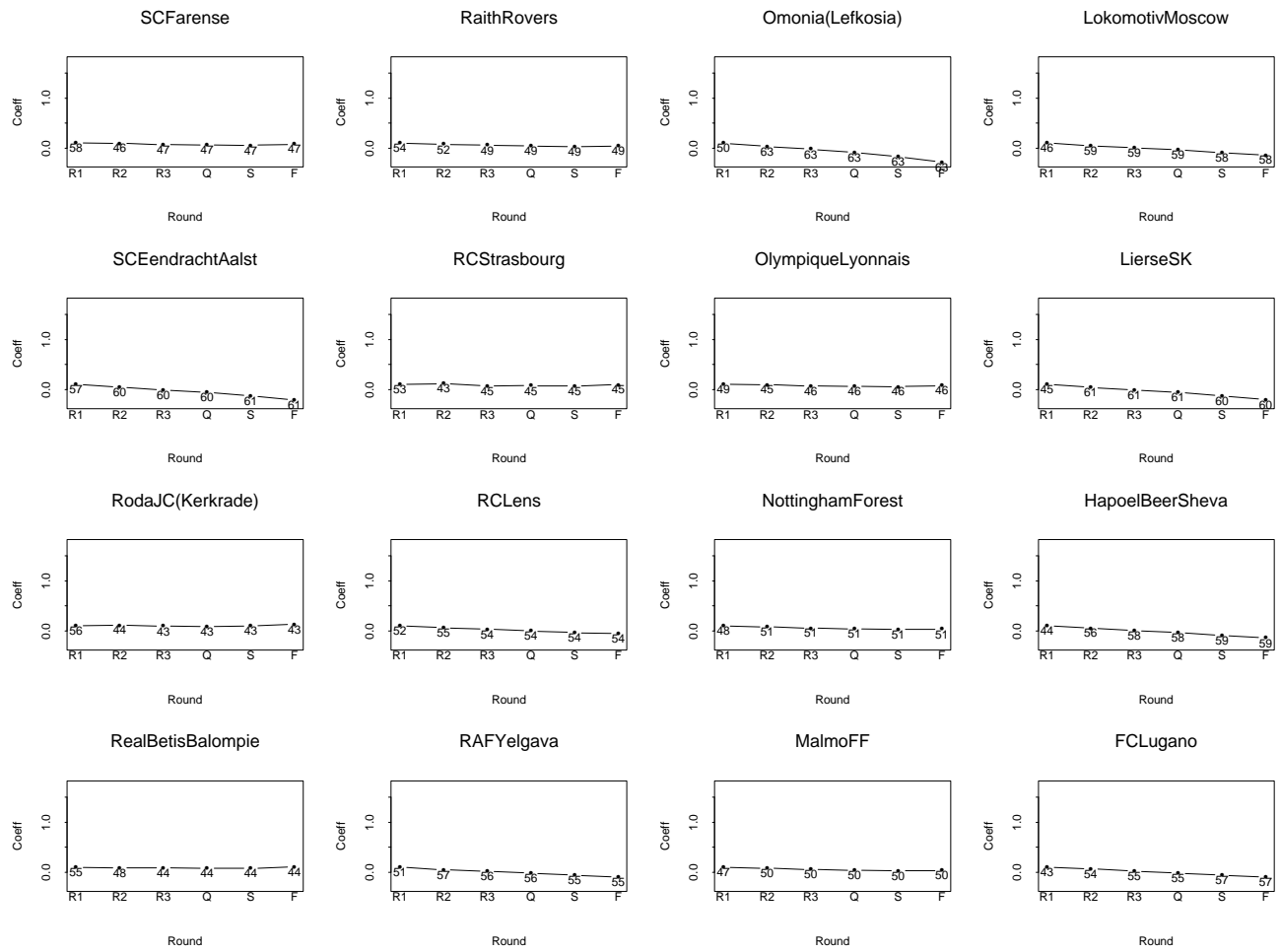


Figure 35 Using all potential opponents (UEFA Cup 1995/96).

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