Matrix-based Methods for College Football Rankings

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1 Introduction

College football season is one of the most popular and anticipated sports competitions in the United States. Many of the National Collegiate Athletic Association (NCAA) Division I-A football games are surrounded by enormous fan interest and receive extensive media coverage. They are attended by tens of thousands of spectators and are followed by millions through the media. As a result, success of a team on the football field brings increased student applications and substantial financial profits to the institution it represents.

Due to these facts, it is especially important that ranking college football teams is as fair and unbiased as possible. However, the format of the NCAA football championship does not allow one to apply traditional ranking methods that are commonly used in professional leagues, where each team plays all other teams during the regular season, and the champion is determined in playoff series. NCAA division I-A includes more than 100 teams, and the number of games played by each team is no more than 15. Clearly, under these conditions, the "quality" of opponents is not the same for different teams, and standard ranking schemes may lead to "unfair" results. Moreover, there are no playoffs in college football, and the national champion is determined in a single game between the #1 and #2 teams in the rankings.

Until several years ago, the rankings were decided purely based on collective opinion of press writers and coaches. Clearly, these ranking principles are not acceptable, since people's opinions are in many cases "biased". For instance, a sports analyst might be impressed by the playing style of a certain team which would affect his decision, moreover, many of those whose votes are considered in the ranking polls (especially, football coaches) cannot see all games of every team during the season and rely on their personal perception or other specialists' judgements. Therefore, this ranking approach can produce "unfair" results. A major controversy took place several times, for example, in 1990, 1991 and 1997 two major polls selected different national champions. In 1998, the Bowl Championship Series (BCS) was introduced as a more trustworthy way of determining who is who in college football. The major components of the current BCS selection scheme are coaches/sports writers polls and computer-based rankings. The BCS system managed to produce an undisputed champion each year since its implementation. However, it is clearly not perfect: it was a general opinion that had Nebraska beaten Miami in 2001 Rose Bowl, the national championship would have to be split between Nebraska and Oregon. Moreover, some of the computer-based rankings included in the BCS scheme use unpublicized methodologies and have been criticized for their poor performance (Kirlin 2002, Martinich 2002).

These facts served as a motivation for many researchers to introduce their own computer-based ranking systems utilizing various mathematical techniques. The proposed approaches include models based on least-squares estimation, linear programming, maximum likelihood estimation, and neural networks (Bassett 1997, Harville 1977, Martinich 2002, Massey 2002, Wilson 1995). These methods take into account various factors and parameters, and they are often too complicated to be understood by people without an appropriate mathematical background. Moreover, in many cases the implementation of these methods is not an easy procedure. The website (Massey 2002) maintains weekly rankings produced by more than 70 different methods.

Plethora of sophisticated ranking systems made the life of ordinary football fans hard, since the rankings produced by different methods may significantly deviate, which means that the performance of their favorite teams may be underestimated or overestimated. Obviously, most of the fans cannot check if a certain ranking system is fair. One can argue that the main goal of any sports tournament (and the ranking system as one of its most important parts) is the fans' satisfaction, therefore, the ranking principles must be *consistent*, but at the same time *explicitly known* and *simple enough* to be understood and reproduced by non-specialists.

As it was pointed out above, the main difficulty one accounters in developing a college football ranking system is the fact that in the NCAA college football tournament the number of games played by every team is very small, and, obviously, one cannot expect the *quality of the opponents* of different teams to be the same. If one tries to rank teams using regular performance measures such as winning percentage, which are suitable for other competitions (for example, NBA, NHL, and MLB, where all teams play each other several times during the season), the results may be inconsistent. Therefore, one of the crucial issues that must be addressed in developing an efficient college football ranking system is taking into account the strength of the opponents of each team.

Another important subject that has been widely discussed and caused controversial opinions is whether the *margin of victory* should be taken into account in the rankings. At the first glance, one can claim that a team that outscores the opponent in a blowout game should stand higher in the rankings than a team who managed to win a close game, and considering score differentials in head-to-head games would provide more accurate rankings. However, several forcible arguments indicate that ranking systems should eliminate the motivation for teams to increase the margin of victory in blowout games, since otherwise it would lead to poor sportsmanship and greatly increase the risk of injuries. One should emphasize that the victory itself, but not the score differential, is the ultimate goal of any sports competition, therefore, the margin of victory should be either not taken into account at all, or limited by a certain (small) amount. Although Martinich (2002) claims that ignoring the margin of victory makes rankings less accurate, in this chapter we will see that it is possible to develop ranking systems that utilize relatively simple principles, take only win–loss information as the input and provide very reasonable results.

Summarizing the above arguments, a "fair" ranking system should

- utilize simple mathematical techniques;
- be available for verifying by non-specialists;
- use win–loss information only (or limit score margins);
- produce reasonable and unbiased results.

In this chapter, we describe two mathematical models for college football rankings that satisfy these criteria to a certain extent. One of these techniques is so-called Colley Matrix Method, which has been recently used as a part of the BCS system. Although the idea of this method is rather simple, it automatically takes into account the schedule strength of each team (while ignoring the margin of victory). This method is presently used as one of the official computer-based rankings in Bowl Championship Series.

Another approach presented here utilizes the *Analytical Hierarchy Process* (AHP), a universal analytic decision making tool used to rank alternatives of various types. This methodology proved to be very efficient in many practical applications, however, it remained unemployed in college football rankings, which can also be treated as ranking the alternatives (*i.e.*, football teams). The AHP method is believed to be a promising college football ranking technique.

Both of these models utilize *matrices* as their main attributes. In particular, the idea of the AHP method is to construct the *comparison matrix* whose elements have certain values determined by the comparison of different pair of alternatives (teams) based on the game outcomes. The principles of constructing this matrix are specifically designed for situations where not all pairs of alternatives can be directly compared, which is exactly the case for a college football tournament.

Numerical experiments presented in the chapter show that despite their simplicity and minimum input information, these approaches yield very reasonable results.

The remainder of this chapter is organized as follows. Section 2 provides the description of the Colley Matrix method for college football rankings. In Section 3 we briefly summarize the main ideas of the Analytic Hierarchy Process methodology, which is then used to develop a college football ranking system. Section 4 presents the results of numerical testing of the described approaches using scores from the last 2 college football seasons (2001-2002). Finally, Section 5 concludes the discussion.

2 Colley Matrix Method for College Football Rankings

One of the well-known mathematical approaches to college football rankings is the *Colley Matrix Method* (Colley 2003), which was recently developed in attempt to produce relatively “fair” and unbiased rankings and is now used as a part of BCS. Among the advantages of this approach one should mention that its main idea is rather simple, which makes this technique easy to understand and implement. Moreover, wins and losses (regardless of score differentials) are the only input information used in the model, which is reasonable due to the arguments presented above. As we will see in this section, the Colley Matrix Method can efficiently take into account the schedule strength of each team, which leads to rather realistic results. Mathematical techniques underlying this ranking system are briefly described below.

Let $n_{w,i}$ be the number of games won by a given team i, and $n_{total,i}$ be the total number of games played by this team. Instead of the winning ratio (defined simply as $n_{w,i}/n_{total,i}$) which is commonly used in practice, a modified quantitative measure of the team's performance is introduced. For any team i, the rating of this team r_i is defined as

$$r_i = \frac{1 + n_{w,i}}{2 + n_{total,i}}. \tag{1}$$

The motivation for this definition is to avoid the values of winning ratios equal to 0 (for the teams with no wins) or 1 (for the teams with no losses), which makes the comparison of such teams inconsistent: for instance, after the opening game of the season the winning team (1 win, 0 losses) is “infinitely better” than the losing team (0 wins, 1 loss). According to Formula 1, the winning team ($r = 2/3$) in this case would have a twice better score than the losing team ($r = 1/3$), which is more reasonable from the practical perspective. Also, note that the default rating of any team with no games played is equal to 1/2, which is the median value between 0 and 1. A win increases the value of r, making it closer to 1, and a loss decreases r towards 0.

After introducing this quantitative performance measure, one needs to adjust it according to the strength of the corresponding opponents. For this purpose the following transformation of the values of n_w is applied. Instead of considering the actual number of wins

$$n_w = \frac{(n_w - n_l)}{2} + \frac{n_{total}}{2} = \frac{(n_w - n_l)}{2} + \sum_{j=1}^{n_{total}} \frac{1}{2},$$

the effective number of wins n_w^{eff} is calculated by adjusting the second term of the above expression, which represents the summation of n_{total} terms equal to 1/2 (index j stands for j-th opponent) corresponding to the default rating of a team with 0 games played. In order to take into account the strength of the opponents, these terms are substituted by actual ratings of the opponent teams r_j, which yields the following formula for the effective number of wins for a given team i:

$$n_{w,i}^{eff} = \frac{(n_{w,i} - n_{l,i})}{2} + \sum_{k=1}^{n_{total,i}} \chi_{ijk} r_j, \tag{2}$$

where $\chi_{ijk} = \begin{cases} 1, \text{if team } i's\ k^{\text{th}} \text{ game was against team} j \\ 0, \text{otherwise.} \end{cases}$

Now, using Formulas (1) and (2), for every team i one can write the following linear equation relating the ratings of this team and its opponents:

$$(2 + n_{total,i}) r_i - \sum_{j=1}^{n_{total,i}} \chi_{ijk} r_j = 1 + \frac{(n_{w,i} - n_{l,i})}{2}. \tag{3}$$

If the total number of teams playing in the NCAA Division I-A tournament is equal to N, then the equations of this form will be written for all N teams, which results in the *linear system* with N equations and N variables. One can rewrite this system in a standard matrix form:

$$C\mathbf{r} = \mathbf{b}, \tag{4}$$

where

$$\mathbf{r} = \begin{bmatrix} r_1 \\ r_2 \\ \cdots \\ r_N \end{bmatrix}$$

represents the vector of variables,

$$\mathbf{b} = \begin{bmatrix} 1 + (n_{w,1} - n_{l,1})/2 \\ 1 + (n_{w,2} - n_{l,2})/2 \\ \cdots \\ 1 + (n_{w,N} - n_{l,N})/2 \end{bmatrix}$$

is the right-hand side vector, and

$$C = [c_{ij}]_{i,j=1...n}$$

is the "Colley matrix", whose elements are defined as follows:

$$c_{ii} = 2 + n_{total,i},$$
$$c_{ij} = -n_{j,i},$$

where $n_{j,i}$ is the number of times the teams i and j played with each other during the season (most commonly equal to 0 or 1).

It turns out that the matrix C has nice mathematical properties, more specifically, it can be proved that it is positive semidefinite (Colley 2003), which enables one to efficiently solve the linear system 4 using standard techniques.

The solution of this system would represent the vector of numbers corresponding to the ratings of all N teams, and the resulting rankings are determined by sorting the elements of the solution vector $\mathbf{r}$ in a decreasing order of their values (*i.e.*, the highest-ranked team corresponds to the largest element in the solution vector, *etc.*).

3 Analytic Hierarchy Process (AHP) Method for College Football Rankings

In this section, we describe the Analytical Hierarchy Process – a powerful decision making technique for ranking alternatives. We first give a brief overview of the AHP methodology, and then apply it to college football rankings.

3.1 Analytic Hierarchy Process: General methodology

The *Analytic Hierarchy Process (AHP)* is a methodology for analytic decision making. It was introduced by Saaty in the late 1970's (Saaty 1977, Saaty 1980), and has been developed into one of the most powerful decision making tools ever since. Golden et al. (1989) describe the AHP as "a method of breaking down a complex, unstructured situation into its component parts; arranging these parts, or variables, into a hierarchic order; assigning numerical values to subjective judgments on the relative importance of each variable; and synthesizing the judgments to determine which variables have the highest priority and should be acted upon to influence the outcome of the situation".

The AHP is applicable to situations involving the comparison of elements which are difficult to quantify. It allows to structure the problem into a hierarchy[3] of simple components. For each of these components, the decision

[3] The word *hierarchy* is from Greek $\iota\epsilon\rho\alpha\ \alpha\rho\chi\eta$, meaning holy origin or holy rule.

maker performs pairwise comparisons of the alternatives which are then used to compute overall priorities for ranking the elements. In the simplest form, the hierarchy used in the AHP consists of three levels (see Figure 1). The goal of the decision is at the highest, first level. Alternatives to be compared are located at the lowest, third level. Finally, the criteria used to evaluate the alternatives are placed at the middle, second level.

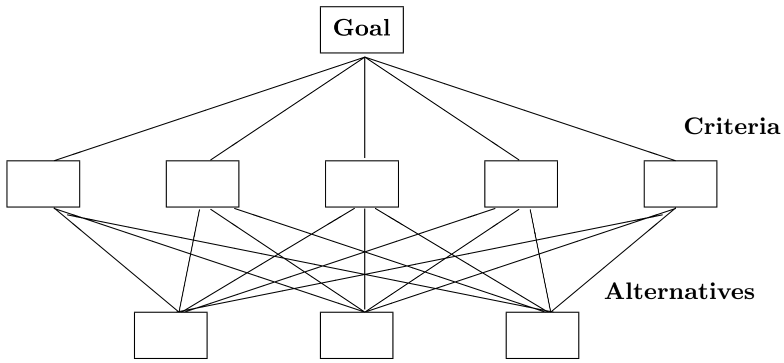

Fig. 1. A three-level hierarchy in AHP.

After defining a hierarchy, the decision maker compares pairs of alternatives using the available criteria and for each compared pair provides a ratio measure which characterizes the relative level of preference of one alternative over the other under the given criterion.

Assume that there are n elements (alternatives, options) to be ranked. As a result of performing pairwise comparisons, a matrix P is created, which is called the *dominance* or *preference matrix*, and whose elements are

$$p_{ij} = \frac{w_i}{w_j},\ i, j = 1, \ldots, n.$$

Here numbers w_i and w_j are used to compare the alternatives i and j. To compare two options, a 10-point scale is often used, in which $w_i,\ i = 1, \ldots, n$ are assigned values from $\{0, 1, 2, \ldots, 9\}$ as follows. If alternatives i and j cannot be compared then $w_i = w_j = 0$. If $i = j$, or i and j are equal alternatives, then $w_i = w_j = 1$. Otherwise,

$$w_i = \begin{cases} 3 & \text{if } i \text{ is} \quad \text{moderately} \\ 5 & \phantom{\text{if } i \text{ is}} \quad \text{strongly} \\ 7 & \phantom{\text{if } i \text{ is}} \quad \text{very strongly} \\ 9 & \phantom{\text{if } i \text{ is}} \quad \text{extremely} \end{cases} \quad \text{preferable over } j.$$

The numbers $2, 4, 6, 8$ are used for levels of preference compromising between two of the specified above. In all of these cases, w_j is set equal to 1. For example, if element i is strongly preferable over element j, we have $p_{ij} = 5$ and $p_{ji} = 1/5$. Zeroes are used when there is no enough information to compare two elements, in which case the diagonal element in each row is increased by the number of zeroes in that row. The above scale is used as an example, however, in general the comparisons could be made using a scale consisting of any set of positive numbers.

The constructed preference matrix is used to derive the n-vector of *priorities* which characterize values of the corresponding alternatives. The larger is the priority value, the higher corresponding alternative is ranked. Given the matrix P, one of the techniques used to derive the vector of priorities and subsequently rank the elements is the following *eigenvector solution*.

Suppose that the vector of priorities $w = [w_i]_{i=1}^n$ is known. Then if we construct the preference matrix and multiply it by w, we obtain

$$Pw = \begin{bmatrix} w_1/w_1 & w_1/w_2 & \cdots & w_1/w_n \\ w_2/w_1 & w_2/w_2 & \cdots & w_2/w_n \\ \vdots & \vdots & \ddots & \vdots \\ w_n/w_1 & w_n/w_2 & \cdots & w_n/w_n \end{bmatrix} \begin{bmatrix} w_1 \\ w_2 \\ \vdots \\ w_n \end{bmatrix} = n \begin{bmatrix} w_1 \\ w_2 \\ \vdots \\ w_n \end{bmatrix}.$$

Therefore, n is an eigenvalue of P with corresponding eigenvector w.

For the comparisons to be consistent, we need to have $p_{ij}p_{jk} = p_{ik}$ for any three alternatives i, j and k. However, in many cases, we can give only estimates of the ratios w_i/w_j, so there may be inconsistencies. In fact, in football the inconsistency and even intransitivity in scores happens quite often when, say team i beats team j, team j beats team k, who in its turn beats team i.

To find an approximation of w, we solve the problem $Pw = \lambda_{max}w$, where λ_{max} is the largest (*principal*) eigenvalue of P, and P is now an estimate of the true preference matrix with $p_{ij} = 1/p_{ji}$ forced (however, this matrix need not be consistent). The solution w is then used as the vector of priorities and the ranking of alternatives is performed as follows. Element i is assigned the value of $w(i)$, and the elements are ranked accordingly to the nonincreasing order of the absolute values of the components of vector w.

A natural question is, how good the obtained ranking is, or how to measure the error appearing as a result of inconsistency? To answer this question, a certain consistency criterion is introduced. It appears that $\lambda_{max} \geq n$ always, and P is consistent if and only if $\lambda_{max} = n$. The *consistency index* (C.I.) of a matrix of comparisons of size $n \times n$ is defined as

$$\text{C.I.} = \frac{\lambda_{max} - n}{n - 1}.$$

The *consistency ratio* (C.R.) is given by C.R. = C.I./R.I., where R.I. is an average *random consistency index* obtained from a sample of randomly generated reciprocal matrices using the corresponding scale. For example, for the aforementioned 0–9 scale, the values of R.I. for $n = 1, \dots, 11$ are given below:

n	1	2	3	4	5	6	7	8	9	10	11	$\cdots$
R.I.	0	0	0.52	0.89	1.11	1.25	1.35	1.40	1.45	1.49	1.51	$\cdots$

The consistency ratio of up to 0.10 is considered acceptable.

Variations of the Analytic Hierarchy Process have been successfully applied to solve complex decision-making problems arising in economics, politics, technology and many other spheres. For more detail on the AHP methodology and its applications the reader is referred to (Golden et al. 1989, Saaty 1980, Saaty and Vargas 1994).

3.2 Application of AHP method to ranking football teams

In this section, we present an approach which can be considered a simple version of AHP for college football rankings. The alternatives in this model are represented by the football teams, and the only criterion used to compare them is outcomes of the games played. Our goal is to rank all the teams based solely on this criterion. To compare two teams, we use a simple three-point scale consisting of 0, 1 and 1.2. Namely, the comparison matrix $P = [p_{ij}]_{i,j=1}^{n}$ is constructed as follows.

- If teams i and j did not play each other, they cannot be compared directly and we assign $p_{ij} = p_{ji} = 0$.
- If $i = j$, or there was a tie between teams i and j, or teams i and j played each other twice within a season with alternative outcomes, then $p_{ij} = p_{ji} = 1$.
- If team i beats team j (once or twice), we set $p_{ij} = 1.2$ and $p_{ji} = 1/1.2$ (*i.e.*, we assume that the winning team is on average 1.2 times better than the loosing team).

Note, that this scale does not take into account the margin of victory, which is in agreement with the arguments presented above.

The next question that arises now is, how realistic are the results obtained by applying the ranking systems described in this chapter? It turns out that the rankings generated by both Colley Matrix method and AHP method are rather reasonable. The next section discusses the results of numerical experiments and compares them with the rankings produced by major press polls.

Table 1. Final rankings comparison: season of 2001.

AP	Team	Record	ESPN	COL	AHP	ARO
1.	Miami Fla	12-0	1	1	1	44.42
2.	Oregon	11-1	2	3	3	46.75
3.	Florida	10-2	3	4	4	38.42
4.	Tennessee	11-2	4	2	2	35.75
5.	Texas	11-2	5	6	9	53.00
6.	Oklahoma	11-2	6	10	10	53.46
7.	LSU	10-3	8	8	6	43.42
8.	Nebraska	11-2	7	5	5	47.85
9.	Colorado	10-3	9	7	8	44.00
10.	Washington St	10-2	11	11	11	49.27
11.	Maryland	10-2	10	13	13	50.42
12.	Illinois	10-2	12	9	7	48.25
13.	South Carolina	9-3	13	15	12	39.09
14.	Syracuse	10-3	14	12	15	47.38
15.	Florida St	8-4	15	16	16	38.25
16.	Stanford	9-3	17	14	14	43.33
17.	Louisville	11-2	16	17	25	69.92
18.	Virginia Tech	8-4	18	31	35	59.08
19.	Washington	8-4	19	18	18	42.00
20.	Michigan	8-4	20	20	17	40.50
21.	Boston College	8-4	23	30	31	55.83
22.	Georgia	8-4	25	24	20	45.50
23.	Toledo	10-2	22	28	43	83.33
24.	Georgia Tech	8-5	26	33	26	41.17
25.	BYU	12-2	24	19	29	76.50
27.	Fresno State	11-3	28	22	24	66.21
30.	North Carolina	8-5	27	25	22	40.77
33.	Arkansas	7-5	33	27	23	33.27
39	UCLA	7-4	41	23	19	40.36
NR	Auburn	7-5	37	26	21	37.75

4 Numerical Experiments

We now present the results of testing the approaches described above using scores from the last two college football seasons (2001–2002). These and other historical scores are available from James Howell's College Football Scores webpage (Howell 2003).

For both methods, only outcomes of the games played within NCAA Division I-A teams are taken into account (*i.e.,* all the games with lower division teams excluded). The results of the experiments are summarized in Tables 4–5. We compare the Colley rankings and AHP rankings that we obtained with Associated Press (AP) writers poll and ESPN/USA Today coaches poll. Only top 25 teams for each of the ranking systems are mentioned. The first column

Table 2. Final rankings comparison: season of 2002.

AP	Team	Record	ESPN	COL	AHP	ARO
1.	Ohio St.	14-0	1	1	1	46.36
2.	Miami	12-1	2	3	4	42.42
3.	Georgia	13-1	3	2	3	40.69
4.	Southern Cal	11-2	4	4	2	28.23
5.	Oklahoma	12-2	5	4	5	47.31
6.	Texas	11-2	7	6	6	48.31
7.	Kansas St.	11-2	6	11	11	52.09
8.	Iowa	11-2	8	8	9	52.77
9.	Michigan	10-3	9	7	7	39.77
10.	Washington St.	10-3	10	12	10	43.42
11.	Alabama	10-3	NR	10	15	48.23
12.	N.C. State	11-3	11	14	12	44.17
13.	Maryland	11-3	13	13	14	46.31
14.	Auburn	9-4	16	19	18	44.42
15.	Boise St.	12-1	12	17	30	81.46
16.	Penn St.	9-4	15	18	19	46.46
17.	Notre Dame	10-3	17	9	8	43.31
18.	Virginia Tech	10-4	14	20	22	52.21
19.	Pittsburgh	9-4	18	24	25	51.31
20.	Colorado	9-5	21	21	16	44.23
21.	Florida St.	9-5	23	15	13	33.79
22.	Virginia	9-5	25	25	17	40.71
23.	TCU	10-2	22	28	32	75.92
24.	Marshall	11-2	19	32	45	83.58
25.	West Virginia	9-4	20	23	21	44.67
26.	Florida	8-5	24	26	23	42.23
27.	Texas Tech	9-5	28	22	20	42.71
32.	South Florida	9-2	29	16	24	59.11

of each table contains a team's AP poll rank, followed by the team's name and season record in columns 2 and 3, respectively. Column 4 contains ranks due to ESPN/USA Today coaches poll. Column 5 presents ranks generated by the Colley Matrix method. Finally, the last two columns contain the ranks obtained using the AHP approach and the average final rank of the opponents (according to the AHP ranking) a team played during the season (ARO).

The first general observation is that both of the considered computer-based rankings are quite consistent with those by the press and coaches.

Another important observation is that both methods find a good balance in reflecting win–loss record and strength of the schedule in the final ranking. It should be noted that both methods adjust the rankings to the schedule strength "internally", *i.e.*, the quality of opponents is measured in terms of ranks of the opponent teams produced by the same ranking system, which excludes a possibility of overestimating or underestimating the schedule strength of any particular team by using some external parameters.

As one can see from the tables, the strength of the schedule is a significant factor in the rankings. For example, Marshall had a good win–loss record in 2002, however, they are only 32^{ns} in the Colley ranking and 45^{th} in the AHP ranking due to the low "quality" of their opponents. On the other hand, despite the 9-5 record, Florida State is significantly higher in the AHP and Colley rankings than in the ESPN ranking because of their tough schedule.

5 Conclusion

In this chapter, we have discussed two matrix-based decision making models for ranking college football teams – Colley Matrix method and Analytical Hierarchy Process method. In both approaches, the information about all games played by every team in the season is summarized in a matrix, however, different principles are used in constructing this matrix.

Clearly, one cannot compare the obtained results with a "true" ranking, because the exact comparison measure between every pair of teams cannot be determined due to obvious reasons. However, as we have seen in this chapter, both methods produce reasonable results and exhibit a high level of agreement with respect to each other, as well as major press polls, which is the best measure of consistency of the rankings. Moreover, both methods are easy to understand and implement, which makes them attractive for practical use.

The Colley Matrix method is already being implemented as a part of the BCS. The AHP ranking method is a promising technique as well. Although in the simplest setup described in this chapter this method uses only wins and losses as input (which is motivated by the arguments presented above), the AHP method is very flexible, and it can be easily adjusted to include other criteria, such as margin of victory, home-field advantage, and experts opinion. However, even in its simplest form, the method produces very reasonable results, as evidenced by our experiments using the scores from the last two seasons. Moreover, variations of the AHP methodology could also be applied to other decision-making problems arising in sports, and in particular in college football. Therefore, we strongly believe that the AHP ranking method proposed in this chapter is practical and can be used in various sports applications to produce fair and unbiased results.

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